ST. HENRY'S COLLEGE KITOVU

A'LEVEL APPLIED MATHEMATICS P425/2 SEMINAR QUESTIONS 2018

STATISTICS AND PROBABILITY

1. a) Events A and B are such that $P(A)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{5}$ and $P(A^1\cap B^1)=\frac{1}{6}$. Find;

   i) $P(A\cup B)$,

   ii) $P(A/B^1)$.

   b) There are 3 black and 2 white balls in each of 2 bags. A ball is taken from the first bag and put in the second bag. A ball is then taken from the second and put in the first bag. What is the probability that there is now the same number of black and white balls in each bag as there were to begin with?

2. a) The chance that Moses wins a game is $\frac{1}{3}$. If he plays nine games in a row, what is the;

   (i) expected number of games,

   (ii) chance of winning at least two games.

   b) At a bottle manufacturing factory, the new machine approximately makes 19% of the bottles that are damaged. If a random sample of 400 bottles is taken, find the probability that;

   (i) more than 31 bottles will be damaged,

   (ii) between 30 and 40 bottles inclusive will be damaged.

3. The table below show the frequency distribution of marks obtained by a group of students in a paper two mathematics examination.

Marks(%)	10 –	20 –	35 –	45 –	65 –	80 –	90 –
Frequency density	1.8	2.4	5.8	3.3	1.2	0.4	0

   a) Calculate the;

   i) modal mark,

   ii) mean mark,

   iii) standard deviation,

   iv) number of students who scored above 54%.

   b) Draw a cumulative frequency curve and use it to estimate the;

   i) P_{10}-P_{60} range,

   ii) number of students who scored below 40%,

   iii) least mark if 20% of the students scored a distinction.

4. The table below shows the cost of ingredients used for making Chapattis for two different birthday parties for 2016 and 2017.

Ingredients	**Cost**	
	2016	**2017**
Salt	200	350
Baking flour	3800	4600
Cooking oil	1500	1800

By taking 2016 as a base year, calculate the price relative for each ingredient and hence, obtain the average index number.

5. The table below shows marks scored by 8 students in mock and UNEB final examinations in Applied Mathematics.

Mock Examination	79	67	52	71	97	55	41	86
UNEB Final Examination	75	60	45	55	85	43	30	70

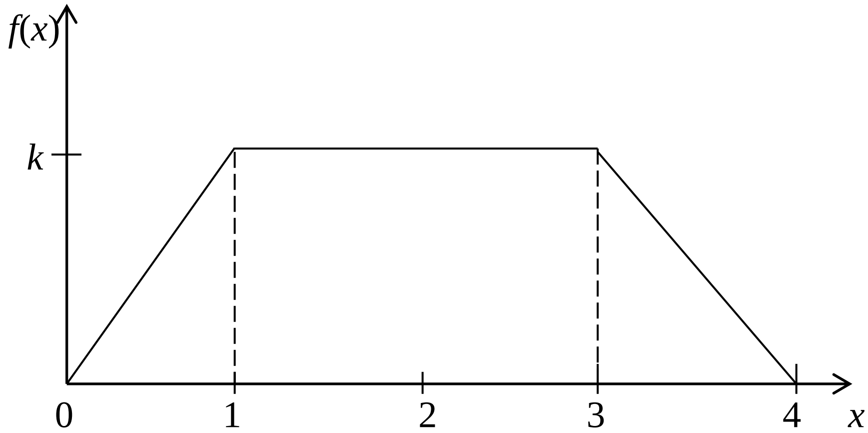
a) (i) Draw a scatter diagram for the data and comment on your result.
(ii) On the same diagram draw a line of best fit.
(iii) Use the line of best fit to estimate the mark that a student who scored 68 in Mock will score in UNEB.

b) Calculate the rank correlation coefficient for the marks in Mock and UNEB and comment on your result.

6. A random variable X takes the integer value x with P(x) defined by
$P(X=1)=P(X=2)=P(X=3)=kx^2, P(X=4)=P(X=5)=P(X=6)=k(7-x)^2$
.
Find the;
a) value of the constant k, hence sketch the graph of $f(x)$.
b) E(Y) and Var(Y) where $Y=4X-2$.

7. The continuous random variable X is such that $X \sim R(a,b)$. The lower quartile is 5 and the upper quartile is 9.
Find;
a) the values of a and b,
b) $P(6<X<7)$,
c) the cumulative distribution function F(x).

8. The probability distribution function of a continuous random variable **X** is represented as shown.



Find the;

(i) mean of X,

(ii) value of k,

(iii) expression for the distribution,

(iv) $P(X \leq x)$, hence the median,

(v) P(1 .8 < x < 3.2).

9. The continuous random variable Y has a cumulative distribution function given by;

$$F(y) = \begin{cases} 0; & y < 1 \\ Ay^2(y^2 - 1); & 1 \leq y \leq 2 \\ 1; & y > 2 \end{cases}$$

Find the:

(a) value of A,

(b) 90^{th} Percentile,

(c) f(y), probability density function of Y,

(d) E(Y)

(e) Var(2Y+3)

10. The positive error made by a machine while manufacturing metal strips is a random variable which can take up any value up to 0.5 cm. it is known that the probability of the length being not more than y centimeters $(0 \leq y \leq 0.5)$ is equal to ky. Determine the;

(a) value of k,

(b) median positive error,

(c) probability distribution function,

(d) expected value of y,

(e) standard deviation correct to 3.s.f.

11. On day one of the Coachella music festival, the height of the revelers can be modeled into a normal distribution of mean 1.75m and variance $0{,}0064m^2$. A draw is to be carried out and it is decided that one should have a height greater than 1.67m but less than 1.83m to participate.

(a) Find the:

(i) percentage of the people who qualify to take part in the draw.

(ii) fraction that is rejected because they are too tall.

By day three of the event, the heights of the people present are normally distributed with mean, μ, and standard deviation, 0.085m. When the criteria used to select participants is not altered, 3.5% of the revelers are rejected because they are too short.

(b) Find the:

(i) value of μ,

(ii) probability that a reveler whose height exceeds the mean qualifies to take part in the draw.

12. The germination time of a certain species of beans is known to be normally distributed. In a given bath of these beans, 20% take more than 6 days to germinate and 10% take less than 4 days.

(a) Determine the mean and standard deviation of the germination time.

(b) Find the 99.5% confidence limits of the germination time.

MECHANICS

13. (a) A particle has an initial position vector of $(4i + 3j - 7k)m$. The particle moves with a constant velocity of $6ms^{-1}$ parallel to $2i - j + 2k$. Find the position vector of the particle $3s$ later. Hence find how far the particle is from the origin.

(b) A particle A of mass 2 kg moves under the action of a constant force $(2i + 4j + 3k)N$. At $t = 0s$, the particles is stationary and at a point with position vector $(4i - 10j + 7k)m$. Find the position vector of the particle at time $t = 5s$.

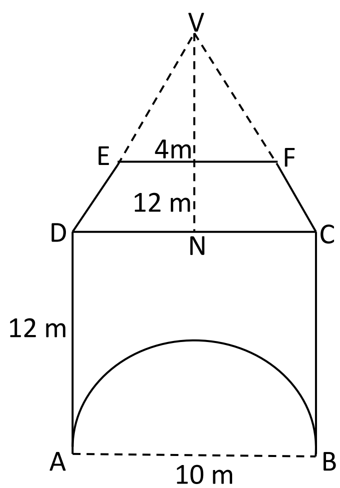
14.(a) An aircraft, at a height of 180m above horizontal ground and flying horizontally with a speed of $70ms^{-1}$, releases emergency supplies. If these supplies are to land at a specific point, at what horizontal distance from these point must the aircraft release them?

(b) A particle is projected from a point on a horizontal plane and has an initial speed of $42ms^{-1}$. If the particle passes through a point above the plane, $70m$ vertically and $60m$ horizontally from the point of projection, find the possible angles of projection.

15.(a) A ball is projected from a point O with an angle of projection α. Find α if the horizontal range of the particle is five times the greatest height reached by it.

(b) A stone thrown upwards from the top of a vertical cliff $56m$ high falls into the sea 4s later, $32m$ from the foot of the cliff. Find the speed and direction of projection. (Take g to be $10ms^{-2}$)

16.



ABCD is a uniform rectangular lamina with AB = $10m$ and AD = $12m$. A semi-circle of diameter AB is cut off. An isosceles uniform triangular lamina of base equal to CD, EF = 4m, NV = 12m and EFV cut off is joined at side CD as shown above.

(a) Calculate the centre of gravity of the remaining system.

(b) If the system is then suspended at A, find the angle side AD makes with the downward vertical.

17. A non-uniform ladder AB of length 10m, weighing $5W$ and centre of mass $4m$ from A rest in a vertical plane with end B against a rough vertical wall and the end A against a rough horizontal surface. If the coefficient of friction at each end is $\frac{1}{4}$ and $\frac{1}{2}$ respectively.

(a) A man of weight $13W$ begins to ascend the ladder from the foot, find how far he will climb before the ladder slips?

(b) If a horizontal inextensible string is the attached from end A to the base of the wall, find the tension in the string when the man climbed the ladder to end B.

18. Two airfields A and B are $150km$ apart with B on a bearing of 045^0 from A. A wind of $30kmh^{-1}$ is blowing from a direction 260^0. Assuming this wind remains constant throughout. Find the time required for aircraft to fly from A to B and back to A again, if the aircraft can fly at $100kmh^{-1}$ in still air.

19. A helicopter sets off from its base and flies at $50ms^{-1}$ to intercept a ship which, when the helicopter sets off, is at a distance of 5km on a bearing of 335^0 from the base. The ship is travelling at $10ms^{-1}$ on a bearing of 095^0. Find;

(a) The course that the helicopter pilot should set if he is to intercept the ship as quickly as possible.

(b) The time interval between the helicopter taking off and it reaching the ship.

20. At 11:45 am, a trawler is 10km due east of a launch. The trawler maintained a steady $10kmh^{-1}$ on a bearing 180^0 and the launch maintains a steady a steady $20kmh^{-1}$ on a bearing 071^0.

(a) Find the minimum distance the boats are apart in the subsequent motion, and the time at which this occurs.

(b) Find to the nearest minutes, the length of time for which the two boats are within 8km of each other.

21. ABCD is a rectangle with AB = $4m$ and AD = $3m$. Forces of $9N$, $8N$, $4N$, $7N$ and $10N$ act along AB, CB, CD, AD and BD respectively with the direction indicated by the order of the letters.

(a) Calculate the magnitude and direction of the single force that could replace this system of forces.

(b) Find the equation of the line of action of the single force and where the line cuts AB.

22.(a) Find the coordinates of centre of gravity of a uniform lamina which lies in the first quadrant and is enclosed by the curves $y = 3x^2,\ y = 4 - x^2$ and the y – axis.

(b) Two rods AB and BC are joined together at B such that $A\hat{B}C = 50^0$. AB is uniform, of length $7m$ and mass $5kg$. BC is uniform, of length 6m and mass $5kg$. Find the distance of the centre of gravity from B.

23.(a) A small object of weight $4w$ in rough contact with a horizontal plane is acted upon by a force inclined at 30^0 to the plane. When the force is of magnitude $2w$, the object is about to slip. Calculate the magnitude of the normal reaction and coefficient of friction between the object and the plane.

(b) A particle of mass $5kg$ is placed on a rough plane which is inclined at 30^0 to the horizontal. The angle of friction between the particle and the plane is 14^0. Find the horizontal force that should be applied to the particle so that;

(i) The particle is just prevented from sliding down the plane.

(ii) The particle moves up the plane with an acceleration of $3.2ms^{-1}$.

24. A body A of mass $2kg$ is moving with a velocity $(-2i + 3j)ms^{-1}$ when it collide with a body B of mass $5kg$, moving with a velocity $(6i - 10j)ms^{-1}$. Immediately after the collision the velocity of A is $(3i - 2j)ms^{-1}$. Find;

(a) The velocity of B after the collision

(b) The loss in kinetic energy of the system due to the collision

(c) The impulse of A on B due to the collision.

25.(a) A particle is attached to one end of a light inextensible string which has its other end attached to a fixed point A. with the string taut, the particle describes a horizontal circle with a constant angular speed 2.8 rad s^{-1}, the centre of the circle being at a point O vertically below A. find the distance OA.

(b) A vehicle is just on the point of slipping when parked on a bent that is banked at an angle of 20^0 to the horizontal.

(i) Find the coefficient of friction between the vehicle tyres and the surface of the road

(ii) If the vehicle were driven around this bend in a horizontal circular path of radius $60m$, find the greatest speed it could attain without slipping occurring.

26. A water pump raises $40kg$ of water a second through a height of $20m$ and ejects it with a speed of $45ms^{-1}$.

(a) Find the kinetic energy and potential energy per second given to the water

(b) Calculate the effective rate at which the pump is working

27. With its engine working at a constant rate of $18k$W, a vehicle of mass 1.5 tones ascents a hill of 1 in 98 against a constant resistance to motion of $450N$. find;

(a) The acceleration of the vehicle up the hill when travelling with a speed of $10ms^{-1}$.

(b) The maximum speed of the vehicle up the hill.

28.(a) A particle performs a SHM of periods 4s and amplitude $2cm$ about a centre O. find the time it takes the particle to travel from O to a point P, a distance $\sqrt{2}\ cm$ from O.

(b) A particle moves with SHM about a mean position O. when passing through two points which are $2m$ and $2.4m$ from O the particle has speeds of $3ms^{-1}$ and $1.4ms^{-1}$ respectively. Find the amplitude of the motion and the greatest speed attained by the particle.

NUMERICAL METHODS

29.(a) Given that $x = 2.40$, $y = 5.613$ and $z = 8.446$, each number rounded off to the given number of decimal places. Find the;

(i) Limits within which the exact value of $\frac{x(4.5-z)}{y}$ lies

(ii) Percentage error made in calculating $\frac{z-y}{x}$. (give your answer correct to 2dps)

(b) Two decimal numbers X and Y were rounded off to give x and y with errors e_x and e_y respectively. Show that the maximum absolute and relative errors made in approximating $XY^{\frac{1}{2}}$ by $xy^{\frac{1}{2}}$ are given by $\left|y^{\frac{1}{2}}e_x\right| + \left|\frac{xe_y}{2y^{\frac{1}{2}}}\right|$ and $\left|\frac{e_x}{x}\right| + \frac{1}{2}\left|\frac{e_y}{y}\right|$ respectively.

30.(a) Use trapezium rule with six ordinates to estimate $\int_0^{\frac{\pi}{3}} tanx\, dx$, correct to **three** decimal places.

(b) Calculate the error made the estimate in (a) above and suggest how that error can be reduced.

31.(a) The table below shows the values of a continuous function f with respect to x.

x	1	2	3	4
$f(x)$	-1.632	-0.865	0.050	1.018

Using linear interpolation or extrapolation, find;

(i) $f(x)$ when $x = 2.7$

(ii) $f^I(1.2)$.

(b) Show that the root of the equation $f(x) = e^{-x} + x - 3$ lies between 2 and 3. Hence use linear interpolation to find the root correct to **two** decimal places

32.(a) Show that the root of the equation $lnx + 2x - 3 = 0$ lies between 1 and 2.

(b) Show that the iterative formula based on Newton Raphson's method for solving the equation is (a) above is given by

$$x_{n+1} = \frac{x_n(4-lnx_n)}{1+2x_n}, \ n = 0, 1, 2, ...$$

Hence find the root of the equation correct to **two** decimal places.

33.(a) Show that the Newton Raphson's formula for finding the fourth root of a number N is

$$x_{n+1} = \frac{1}{4}\left(3x_n + \frac{N}{x_n^3}\right), \; n = 0, 1, 2, \dots$$

(b) Construct a flow chart that;

- Read N and the first approximation x_0
- Compute and print the root x_{n+1} and the number of iteration, n.

Using the flow chart, show that $(67)^{\frac{1}{4}} \approx 2.86$ to two decimal places.

END

STATISTICS AND PROBABILITY

1. (a) The data below was obtained from a survey carried out on the temperature variations of 10 cities in a cold season of the year.

$0^oC, -2^oC, 3^oC, 4^oC, -4^oC, -10^oC, 5^oC, 1^oC, -8^oC, -7^oC.$

Determine the;

(i) mean temperature,

(ii) variance.

(b) A sample of **n** members of the rotary club of Masaka was asked how many crates of beer they took in a given month.
The results were as follows $\sum x = 225, \sum x^2 = 1755$. Find the possible values of **n** if the standard deviation is **1.5** and hence find the respective mean number of crates taken by the Rotarians.

2. The table shows the marks scored by a group of candidates in a mathematics exam.

Marks (%)	10 - 19	20 - 29	30 - 34	35- 44	45-54	55-64	65-69
Frequency density	0.7	2.6	4.2	3.8	4.6	2.8	2.6

(a) Draw a histogram and use it to estimate the modal mark.

(b) Calculate the;

(i) mean mark,

(ii) median mark,

(iii) standard deviation.

3. The table below shows the marks obtained by a group of students in a math test.

Marks(%)	$20-29$	$30-34$	$35-44$	$45-64$	$65-74$	$75-84$
Frequency	5	5	12	20	10	8

Calculate the;

(a) mean mark,

(b) standard deviation,

(c) mode,

(d) median,

(e) semi-interquartile range,

(f) middle 60% of the marks.

(g) number of students whose marks exceed 47%.

4. The table below shows the heights (y) in centimetres and the age (x), in years, of a group of students in a certain secondary school.

Student	A	B	C	D	E	F	G	H	I	J	K	L
Age (x)	12	14	13	15	17	20	17	15	18	19	14	16
Height (y)	130	136	120	120	153	160	155	142	145	172	140	157

(a) Construct a scatter diagram, draw the line of best fit and comment hence estimate x when y = 142.

(b) Giving rank 1 to the tallest student and oldest student, calculate the rank correlation coefficient and comment at 5% level of significance.

5. A class of 10 students were examined in Economics (E) and Mathematics (M). The following table shows their scores out 10.

Student	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
E	7.2	5.2	3.1	3.8	8.1	5.2	4.0	6.0	6.3	7.5
M	6.4	6.0	6.5	4.3	7.0	4.8	3.4	6.2	5.9	6.0

(a) Plot the above on a scatter diagram.

(b) Draw a line of best fit and estimate the value of E when M = 8.6.

(c) Compute the rank correlation coefficient between E and M and comment on you result.

6. (a) The table below shows the expenditure (in Ug. shs) of a student during the first and second terms.

ITEM	EXPENDITURE		WEIGHT
	First term	Second term	
Clothing	46,500	49,350	5
Pocket money	55,200	57,500	3
Books	80,000	97,500	8

Using first term expenditure as the base, calculate the average weighted price index to **one** decimal place.

(b) The price relative of commodity in 2010, using 2009 as base year was 105. The price relative of same commodity in 2012 using 2010 as base year was 95. Given that the cost of the commodity in 2009 was shs. 259,250, find its cost in 2012.

(c) 2015 being the base year, the price index of a particular commodity in 2017 was 110 and if 2017 is used as the base year, the price index in 2018 is 120. Calculate the index number for 2018 taking 2015 as the base year.

7. (a) Two events A and B are such that $P(A) = 0.7$, $P(A \cap B) = 0.45$ and $P(A^1 \cap B^1) = 0.18$. Find;

   (i) $P(B')$ (ii) $P(AorB)$

   (b) M and N are two events such that $P(M) = 0.3$, $P(N) = 0.1$ and $P(M/N) = 0.2$, find:

   (i) $P(M' \cup N')$, (ii) $P(M/N')$

   (c) Events A, B and C are mutually exclusive and exhaustive such that $P(A) = k(1-2x), P(B) = k(1+x)$ and $P(C) = k(3+x)$. Find the value of

   (i) k (ii) $P(B)$ if $x = \frac{1}{4}$.

   (d) If two events C and D independent such that their chance of occurring together is $\frac{1}{5}$ and the chance that either C or D occurs is $\frac{7}{8}$.

   (i) Show that C' and D' are also independent.

   (ii) Find $P(\mathrm{C})$ and $P(\mathrm{D})$.

8. (a) A bag A contains 5 red ball and 3 black balls and bag B contains 3 red balls and a black ball. A bag is selected at random and two balls drawn from it without replacement. If A is twice as likely to be chosen as B,

   (i) Find the probability that both balls are of different colours and the probability that the balls are from A given that they are of different colours.

   (ii) Construct a probability distribution table for the number of black balls drawn. Hence find the mean number of black ball drawn and the variance.

   (b) A box P contains 1 red, 3 green and 1 blue bead. Box Q contains 2 red, 1 green and 2 blue beads. A balanced die is thrown and if the throw shows a six, box P is chosen otherwise box Q is chosen. A bead is drawn at random from the chosen box. Given that a green bead is drawn, find the probability that it came from box P.

9. A random variable X has a p.d.f given by;

$$f(x) = kx; \quad 0 \le x \le 3, \quad f(x) = 3k(4-x); \quad 3 \le x \le 4$$

$$f(x) = 0; \ x < 0 \ and \ x > 4.$$

   (a) Sketch $f(x)$ hence the value of k.

   (b) Find $E(3X - 1)$.

   (c) Obtain the cumulative distribution, $P(X \le x)$. Hence find

   (i) $P(X < 3.5)$, (ii) the median and inter quartile range.

10.(a) The continuous random variable Y is uniformly distributed in the interval $a < Y < b$. The lower quartile is 5 and the upper quartile is 9. Find

(i) the values of a and b, (ii) $P(6 < X < 7)$

(iii) the cumulative distribution function F(x).

(b) Given that $X \sim R[32, 37]$, find;

(i) E(X)

(ii) the probability that X lies within one standard deviation of the mean.

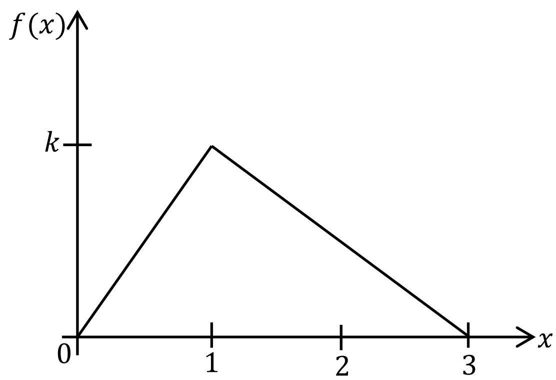
(c) A continuous random variable X has cumulative distribution function;

$F(\text{X}) = \frac{x-2}{5},\ 2 \leq \text{X} \leq 7$

Find

(i) $P(\text{X} < 6/\text{X} > 3)$ (ii) $E(\text{X})$ and $Var(\text{X})$,

11. A random variable X has its probability function as shown below.



(i) Find the value of k.

(ii) Find the probability density function, $f(x)$.

(iii) Compute $P(\left| X - 1 \right| < \frac{1}{2})$

(iv) Derive the distribution function, F(x) and hence find the median of X.

12. (a) A couple is equally likely to produce a boy or a girl. Find the probability that in a family of five children there more boys than girls.

(b) A box contains equal number of red counters as yellow counters. A counter is taken from the box, its colour is noted the replaced. This is performed eight times in all. Calculate the probability that;

(i) exactly three will be red,

(ii) at least one will be red,

(iii) more than four will be yellow.

(c) A random variable X is such that $X \sim B(10, p)$ where $p < 0.5$ and $Var(X) = 1.875$. Find

(i) the value of p,

(ii) E(X),

(iii) $P(X = 2)$.

13. The masses of packets of sugar from a certain factory are normally distributed. In a large consignment of packets of sugar, it is found that 5% of them have a mass greater than 510 g and 2% have a mass greater than 515 g.

Estimate;

(a) the mean and standard deviation of this distribution,

(b) the probability that a packet picked at random from this consignment weighs more than 500 g.

14. The records from a Health Centre IV in Wakiso district showed that 80% of the patients who visited the centre on a certain day had malaria. Find the probability that on a particular day when 200 patients visited the centre;

(a) more than 70 patients tested positive for malaria,

(b) at least 55 patients were not suffering from malaria,

(c) less than 170 patients tested positive for malaria.

15. The heights of boys in a certain village follow a normal distribution with mean 150 cm and variance 25 cm^2. Find the probability that a boy picked at random from the village has height:

(a) less the 153 cm,

(b) more than 158 cm,

(c) between 149 cm and 159 cm,

(d) more than 10 cm difference from the mean height.

16. The masses of cows on Mr. Kato's farm of local cattle are normally distributed. It is discovered that 5% of the cows have a mass greater than 110 kg and 2% have a greater than 115 kg. Estimate the:
 (a) mean and standard deviation of this distribution,
 (b) number of cows in a group of 1000 cows which weigh between 109 kg and 121 kg.
 A veterinary doctor visited the farm and found that 10% of the cows malnourished. Determine the weight of the heaviest malnourished cow.

17.(a) The chance that a hen on Mrs. Musoke's poultry farm is infected with a deadly virus is 0.4. If a sample of 150 hens were inspected on the farm, find the 99.5% confidence limits for the mean number of cows that are infected.
 (b) A 95% confidence interval for the mean life of a particular type of smartphone battery was calculated and the confidence limits were 1023.3 hours and 1101.7 hours. The interval was based on a sample of 36 smartphone batteries. Find the 99% confidence interval for the mean life of this type of batteries.
 (c) A random variable of 50 readings taken from a normal population gave the following data: $\sum x = 163$ and $\sum x^2 = 548$. Calculate the:
 (i) unbiased estimate for the population variance,
 (ii) 98.46% confidence interval for the population mean.

MECHANICS

18.(a) ABC is an isosceles triangle, right angled at A with $\overline{AB} = 2$m. forces of 8N, 4N, and 6N act along the sides BA, CB, and CA respectively. Find the magnitude and direction of the resultant force.
 (b) In a square ABCD, three forces of magnitude 4N, 10N, and 7N act along AB, AD and CA respectively. Their directions are in the order of the letters. Find the magnitude of the resultant force.
 (c) In an equilateral triangle PQR, three forces of magnitude 5N, 10N, and 8N act along sides PQ, QR, and PR respectively. Their directions are in order of the letters. Find the magnitude of the resultant force.

19. A particle of mass 15kg is pulled up a smooth slope by a light inextensible string parallel to the slope. The slope is 10.5 m long and inclined at $\sin^{-1}(4/7)$ to the horizontal. The acceleration of the particle is $0.98ms^{-2}$.
Determine the:

(a) Tension in the string.

(b) Work done against gravity when the particle reaches the end of the slope

21. (a) A particle performs a SHM of periods 4s and amplitude $2cm$ about a centre O. find the time it takes the particle to travel from O to a point P, a distance $\sqrt{2}\ cm$ from O.

(b) A particle is moving with linear simple harmonic motion of amplitude 1.5m. The speed of the particle is $\sqrt{50}$ ms^{-1} when its displacement from the end point is 1m. Calculate its maximum acceleration.

(c) A particle moves with SHM about a mean position O. when passing through two points which are $2m$ and $2.4m$ from O the particle has speeds of $3ms^{-1}$ and $1.4ms^{-1}$ respectively. Find the amplitude of the motion and the greatest speed attained by the particle.

22.(a) A body moving initially with a velocity u covers a distance x after t seconds. If it moves with a uniform acceleration a, derive an expression relating x, t, u, and a.

(b) A train approaching a station does two successive half kilometers in 16s and 20s respectively. Assuming a uniform retardation, calculate the further distance the train runs before it comes to rest.

(c) A body falls from rest from the top of a tower and during the last second it falls $\frac{9}{25}$ of the whole distance. Find the height of the tower.

23. A body moving with acceleration $e^{2t}\boldsymbol{i} - 3\sin 2t\boldsymbol{j} + 4\cos 2t\,\boldsymbol{k}$ is initially located at the point (1, -2, 2)m and has a velocity of $4\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ ms^{-1}. Find the;

a) Speed of the body when $t = \frac{\pi}{4}$s.

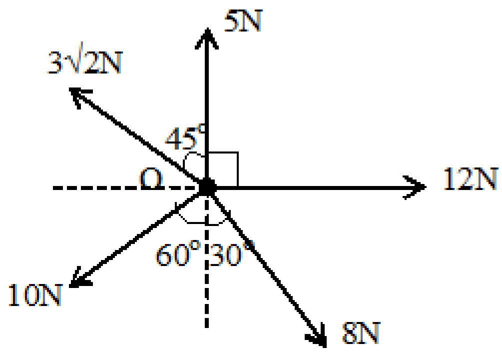
b) Distance of the body from the origin at $t = \frac{\pi}{4}$

24. (a) Forces of 7N and 4N act away from a common point and make an angle of θ^o with each other. Given that the magnitude of their resultant is 10.75N, find the;

(i) Value of θ^o

(ii) Direction of the resultant force

(b) In the diagram below find the magnitude and direction of the resultant force



25. A boy throws a stone at a vertical wall a distance, d away. Given that R is the maximum range on the horizontal through the point of projection that can be attained by the speed of projection, show that;

(a) the height above the point of projection of highest point on the wall he can hit is $\left(\frac{R^2-d^2}{2R}\right)$,

(b) in this case, the angle of projection is $tan^{-1}\left(\frac{R}{d}\right)$.

26. A ball is hit at a point O, which is at a height of 2m above the ground and at a horizontal distance 4m from the wall, the initial speed being in a direction of 45^0 above the horizontal. If the ball just clears the wall which is 1m high,

(a) show that the equation of path of the ball is $16y = 16x - 5x^2$.

(b) calculate the;

(i) distance from the net at which the ball strikes the ground.

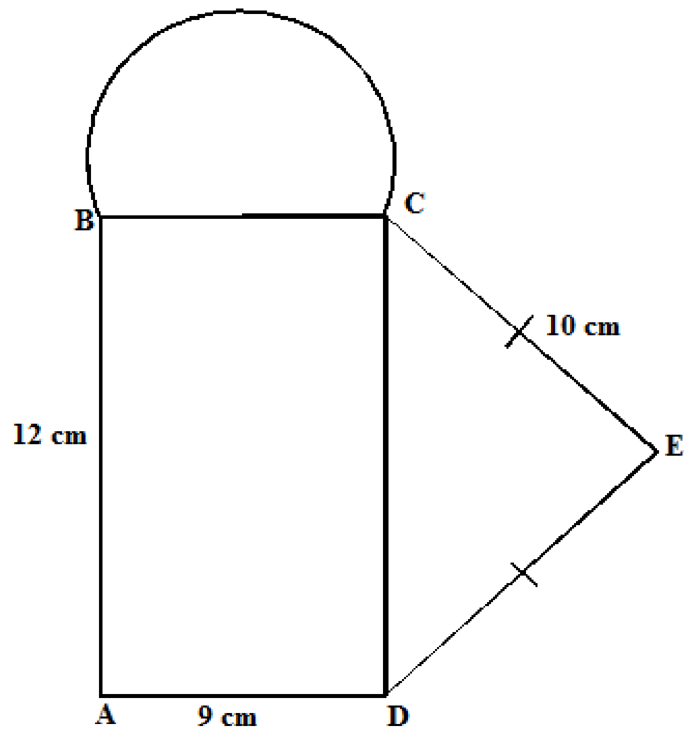
(ii) magnitude and direction of the velocity with which the ball strikes the ground.

27. With its engine working at a constant rate of $18kW$, a vehicle of mass 1.5 tones ascents a hill of 1 in 98 against a constant resistance to motion of $450N$. find;

(a) The acceleration of the vehicle up the hill when travelling with a speed of $10ms^{-1}$.

(b) The maximum speed of the vehicle up the hill.

28. The figure below represents a lamina formed by welding together rectangular, semi-circular and triangular metal sheets.



Find the position of the centre of gravity of the lamina from the sides AB and AD.

29. At 10:00 am, ship A and ship B are 16 km apart. Ship A is on a bearing N35^{o}E from ship B. ship A is travelling at 14 kmh^{-1} on bearing S29^{o}E. ship B is travelling at 17 kmh^{-1} on a bearing N50^{o}E. determine the;

(a) Velocity of ship B relative to ship A

(b) Closest distance between the two ships and the time when it occurs

30. (a) Two ships A and B are observed from a coast guard station and have the following displacement velocities and times.

Ship	Displacement	Velocity	Time (t)
A	$(\boldsymbol{i} + 3\boldsymbol{j})$ km	$(\boldsymbol{i} + 2\boldsymbol{j}) kmhr^{-1}$	12:00 hours
B	$(\boldsymbol{i} + 2\boldsymbol{j})$ km	$(5\boldsymbol{i} + 6\boldsymbol{j}) kmhr^{-1}$	13:00 hours

Find the time when the two are closest to each other.

(b) If at 13:00 hours ship A changed it's velocity to $\left(\frac{\mathbf{11}}{\mathbf{3}}\boldsymbol{i} + 2\boldsymbol{j}\right) kmhr^{-1}$, show that they collide and find the time and position of collision.

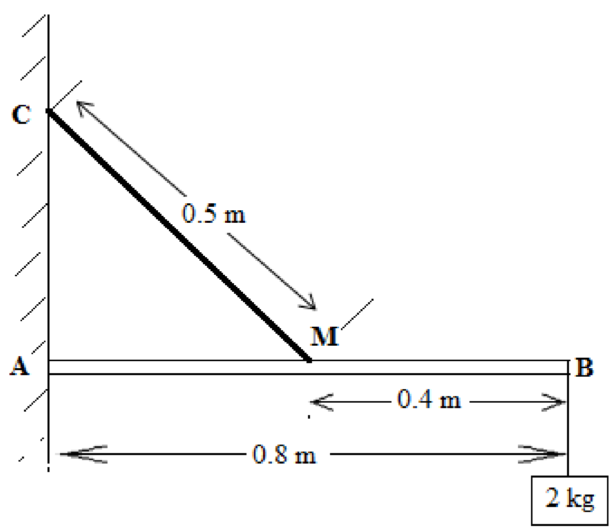
31. A light inextensible string has one end attached to a ceiling, the string passes under a smooth moveable pulley of mass 2 kg and then over a smooth fixed pulley, the particle of mass 5 kg is attached at the free end of the string, the sections of the strings not in contact with the pulleys are vertical, if the system is released from rest and moves in a vertical plane, determine the;

(a) Accelerations of the 2kg and 5kg masses

(b) Tensions of the 2kg and 5kg masses

(c) Distance moved by the system in 1.5 seconds.

32. The figure below shows a uniform beam of length 0.8 metres and mass 1 kg. The beam is hinged at A and has a load of mass 2 kg attached at B.



The beam is held in a horizontal position by a light inextensible string of length 0.5 metres. The string joins the mid-point M of the beam to a point C vertically above A. Find the:

(a) Tension in the string.

(b) Magnitude and direction of the force exerted by the hinge.

33. A non-uniform ladder AB of length 10m, weighing $5W$ and centre of mass $4m$ from A rest in a vertical plane with end B against a rough vertical wall and the end A against a rough horizontal surface. The angle between the ladder and the horizontal is 50° and the coefficient of friction at each end is $\frac{1}{4}$ and $\frac{1}{2}$ respectively.

(a) A man of weight $13W$ begins to ascend the ladder from the foot, find how far he will climb before the ladder slips?

(b) If a horizontal inextensible string is the attached from end A to the base of the wall, find the tension in the string when the man climbed the ladder to end B.

34. A particle of mass 2 kg is acted upon by a force of 21N in the direction $2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$. Find in vector form the;

(a) Force

(b) Acceleration hence its magnitude.

35. Five forces of magnitudes 3N, 4N, 4N, 3N and 5N act along the lines AB, BC, CD, DA, and AC respectively, of a square ABCD of side 1m. The direction of the forces is given by the order of the letters. Taking AB and AD as reference axes; find the

(a) Magnitude and direction of the resultant force.

(b) Equation of the line of action of the resultant force and hence find the point where the resultant force cuts the side AB.

36. A particle of mass 4 kg starts from rest at a point $(2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k})$ m. it moves with acceleration $\boldsymbol{a} = (4\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k})ms^{-2}$ when a constant force $\boldsymbol{F}$ acts on it. Find the:

(a) Force $\boldsymbol{F}$.

(b) velocity at any time t.

(c) work done by the force **F** after 6 seconds

35. At **10.00am**, ship **A** moving at **20km/hr** due east is **10km** South East of another ship ***B***. If ***B*** is moving at **14km/hr** in direction **S30^{0}W** and the ships maintain their velocities, find the;

(a) Time when the ships are **closest** together and the **shortest distance** between the ships.

(b) Bearing of **A** from ***B*** at that time.

36. A particle A initially at the point with position vector $2\mathbf{i} - 5\mathbf{j} + \mathbf{k}\ km$ is moving with a constant velocity of $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}\ kmh^{-1}$. At the same instant, a particle B at the point $(3, 3, 2)$ is moving with a constant velocity of $3\mathbf{i} - 2\mathbf{k}\ kmh^{-1}$. Find the:

(i) relative velocity of particle A to B.

(ii) relative displacement of particle A to particle B at any instant.

(iii) shortest distance between the two particles in their subsequent motion.

37. (a) Two ships A and B are observed from a coast guard station and have the following displacement velocities and times.

Ship	Displacement	Velocity	Time (t)
A	$(\boldsymbol{i} + 3\boldsymbol{j})$ km	$(\boldsymbol{i} + 2\boldsymbol{j})kmhr^{-1}$	12:00 hours
B	$(\boldsymbol{i} + 2\boldsymbol{j})$ km	$(5\boldsymbol{i} + 6\boldsymbol{j})kmhr^{-1}$	13:00 hours

Find the time when the two are closest to each other.

(b) If at 13:00 hours ship A changed it's velocity to $\left(\frac{11}{3}\boldsymbol{i} + 2\boldsymbol{j}\right) kmhr^{-1}$**,** show that they collide and find the time and position of collision.

NUMERICAL METHODS

38. (a) Show that the iterative formula based on Newton Raphson's method for solving the equation $e^{2x} + 4x = 5$ is given by,

$$x_{n+1} = \frac{e^{2x_n}(2x_n - 1) + 5}{2e^{2x_n} + 4}, \text{ n = 0, 2, 3...}$$

(b) (i) Construct a flow chart that;

- reads the initial approximation x_0,
- computes, using the iterative formula in (a) and prints the root of the equation $e^{2x} + 4x - 5 = 0$, and the number of iterations when the error is less than 1.0 x 10^{-4}.

(ii) Perform a dry run of the flow chart when $x_0 = 0.5$.

39. The iterative formulae below are used for calculating the positive root of the $f(x) = 0$.

$$A: \ x_{n+1} = \frac{1}{3}\left(\frac{2x_n{}^3+12}{x_n{}^2}\right)$$

$$B: x_{n+1} = \sqrt{\left(\frac{x_n{}^3+12}{2x_n}\right)}$$

(a) Taking $x = 2$, use each formula twice and hence deduce the most suitable for solving $f(x) = 0$.

(b) Find the root of the equation $f(x) = 0$, correct to three decimal places.

(c) Find the equation whose root is in b) above.

40. (a) Show that the equation $\ln x = \sin x + 2$ has a root between $x = 3$ and $x = 4$. Use linear interpolation to estimate the initial approximation x_o to 1 decimal place.

(b) Using the x_o above and the Newton Raphson method find the root correct it 3 decimal places.

41. (a) The table below shows the values of x and their corresponding natural logarithm

x	5.0	5.2	5.4	5.7	6.0
Inx	1.609	1.647	1.686	1.740	1.792

Use linear interpolation or extrapolation to find

(i) $In\,(5.56)$,

(ii) $e^{1.575}$.

(b) A car consumed fuel amounting to shs 14,800, shs 15,600, shs 16,400 and shs 17,200 in covering distances of 10km, 20km, 30km and 40km respectively. Estimate the;

(i) cost of fuel consumed for a distance of 45km,

(ii) distance travelled if fuel of shs 16,000 is used.

42. (a) Two positive decimal numbers X and Y were approximated with errors E_1 and E_2 respectively. Show that the maximum possible relative error in the approximation of the product X^3Y^2 is $3\left|\frac{E_1}{X}\right| + 2\left|\frac{E_2}{Y}\right|$.

(b) Given that X = 5.64 and Y = 10.0, rounded off to the given number of decimal places, find the;

(i) maximum possible errors in X and Y,

(ii) percentage error made in the approximation of X^3Y^2.

43.(a) Given that $y = xsinx$ and $x = 2$, find the absolute error in y giving your answer correct to 3 significant figures.

(b) The numbers $x = 1.5, y = -2.85$ and $z = 10.345$ were all rounded off to the given number of decimal places. Find the range within which the exact value of $\frac{1}{x} - \frac{1}{y} + \frac{y}{xz}$ lies.

44. (a) Using trapezium rule with five strips evaluate $\int_3^4 \frac{1}{\sqrt{(x-1)^2-3}}dx$, correct to three decimal places.

(b) Find the exact value of $\int_3^4 \frac{1}{\sqrt{(x-1)^2-3}}dx$.

(c) Find the percentage error in the approximation in a) above and suggest how this error can be reduced.

END

ST. HENRY'S COLLEGE KITOVU

A'LEVEL PURE MATHEMATICS P425/1 SEMINAR QUESTIONS 2019

ALGEBRA

1. Solve the simultaneous equations:
   (a) $x^2 + y^2 = 5$, $\frac{1}{x^2} + \frac{1}{y^2} = \frac{5}{4}$
   (b) $\frac{x}{y} + \frac{y}{x} = \frac{17}{4}$, $x^2 - 4xy + y^2 = 1$

2. Find the range of values of x for which
   (a) $\frac{2x+1}{x+2} > \frac{1}{2}$.
   (b) $|2x+1| > 7$

3. Resolve into partial fractions
   (a) $\frac{x^3 + x^2 + 4x}{x^2 + x - 2}$
   (b) $\frac{3x^2 + 8x + 13}{(x-1)(x^2 + 2x + 5)}$
   (c) $\frac{2x^3 + 2x^2 + 2}{(x+1)^2(x^2 + 1)}$

4. Solve the following equations:
   (a) $2^{3x+1} = 5^{x+1}$
   (b) $9^x - 4(3^x) + 3 = 0$
   (c) $\log_x 9 + \log_{x^2} 3 = 2.5$
   (d) $\sqrt{2x-1} - \sqrt{x-1} = 1$
   (e) $\begin{aligned} 2x + 3y + 4z &= 8 \\ 3x - 2y - 3z &= -2 \\ 5x + 4y + 2z &= 3 \end{aligned}$

5. Find:
   (a) The three numbers in arithmetic progression such that their sum is 27 and their product is 504
   (b) The three numbers in a geometrical progression such that their sum 39 and their product is 729.
   (c) The sum of the last three terms of a geometrical progression having n terms is 1024 times the sum of the first three terms of the progression. If the third term is 5, find the last term.

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(2) Mr. Ssentongo Leornard – 0782318244/701503577

(d) Prove by induction that $1^3+2^3+....+n^3=\frac{1}{4}n^2(n+1)^2$ and deduce that

$1^3+3^3+5^3....+(2n+1)^3=(n+1)^2(2n^2+4n+1)$

6. Expand:

(a) $\frac{7+x}{(1+x)(1+x^2)}$ in ascending powers of x as far as the term in x^4.

(b) $\left(1-\frac{3}{2}x-x^2\right)^5$ in ascending powers of x as far as the term in x^4.

(c) Find the term independent of x in the expansion of $\left(2x+{}^1\!/_{x^2}\right)^{12}$ in descending powers of x and find the greatest term in the expansion when $x=\frac{2}{3}$.

(d) Find by binomial theorem, the coefficient of x^8 in the expansion $\left(3-5x^2\right)^{1/2}$ in ascending powers of x.

(e) In the binomial expansion of $(1+x)^{n+1}$, n being an integer greater than two, the coefficient of x^4 is six times the coefficient of x^2 in the expansion $(1+x)^{n-1}$. Determine the value of n.

7. (a) Without using the calculator, simplify $\frac{\left(\cos\left(\frac{\pi}{9}\right)+i\sin\left(\frac{\pi}{9}\right)\right)^4}{\left(\cos\left(\frac{\pi}{9}\right)-i\sin\left(\frac{\pi}{9}\right)\right)^5}$

(b) In a quadratic equation $z^2+(p+iq)z+3i=0$. p and q are real constants. Given that the sum of the squares of the roots is 8. Find all possible pairs of values of p and q.

8. (a) How many different arrangements of letters can be made by using all the letters in the word contact? In how many of these arrangements are the vowels separated?
(b) In how many ways can a team of eleven be picked from fifteen possible players.

9. (a) If α and β are the roots of the equation $x^2-px+q=0$, form the equation whose roots are $\frac{\alpha}{\beta^2}$ and $\frac{\beta}{\alpha^2}$.

(b) If α and β are the roots of the equation $x^2+bx+c=0$, form the equation whose roots are $\frac{1}{\beta^3}$ and $\frac{1}{\alpha^3}$. If in the equation above $\alpha\beta^2=1$, prove that $a^3+c^3+abc=0$

10. (a) If $z=x+iy$ and $\bar{z}$ is the conjugate of z, find the values of x and y such that $\frac{1}{z}+\frac{2}{\bar{z}}=1+i$

(b) If x, y, a and b are real numbers and if $x+iy=\frac{a}{b+\cos\theta+i\sin\theta}$. Show that

$(b^2-1)(x^2+y^2)+a^2=2abx$.

(c) If n is an integer and $z=cos\theta+sin\theta$, show that $2\cos n\theta=z^n+\frac{1}{z^n}$, $2i\sin n\theta=z^n-\frac{1}{z^n}$.

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(2) Mr. Ssentongo Leornard – 0782318244/701503577

Use the result to establish the formula $8cos^4\theta = cos4\theta + 4cos2\theta + 3$.

(e) If z is a complex number and $\left|\frac{z-1}{z+1}\right| = 2$, find the equation of the curve in the Argand diagram on which the point representing z lies.

TRIGONOMETRY

11. If $sin\theta + sin\beta = a$ and $cos\theta + cos\beta = b$, show that $\cos^2\left(\frac{\theta-\beta}{2}\right) = \frac{1}{4}\left(a^2+b^2\right)$

12. Show that $\sin 7x + \sin x - 2\sin 2x \cos 3x = 4\cos^3 3x$

13. If A, B and C are angles of a triangle, show that:

(i) $\cos A + \cos(B-C) = 2\sin B \sin C$

(ii) $\cos\frac{C}{2} + \sin\frac{A-B}{2} = 2\sin\frac{A}{2}\cos\frac{B}{2}$

14. Express $y = 8\cos x + 6\sin x$ in form of $R\cos(x-\alpha)$ where R is positive and α is acute. Hence find the maximum and minimum values of $\frac{1}{8\cos x + 6\sin x + 15}$ and the corresponding angle respectively.

15. Show that:

(a) $\tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

(b) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(c) Find x if $\tan^{-1} x + \tan^{-1}(1-x) = \tan^{-1}\left(\frac{4}{3}\right)$

16. (a) Show that $\cos 4\theta = \frac{\tan^4\theta - 6\tan^2\theta + 1}{\tan^4\theta + 2\tan^2\theta + 1}$

(b) Solve the equation $8\cos^4 x - 10\cos^2 x + 2 = 0$ for x in the range of $0° \leq x \leq 180°$

17. (a) If $\tan\theta = \frac{1}{p}$ and $\tan\beta = \frac{1}{q}$ and $pq = 2$p, show that $\tan(\theta+\beta) = p+q$

(b) Show that $\sin 2A + \cos 2A = \frac{(1+\tan A)^2 - 2\tan^2 A}{1+\tan^2 A}$

18. If α, β and γ are all greater than $\frac{\pi}{2}$ and less than 2π and $\sin\alpha = \frac{1}{2}$, $\tan\beta = \sqrt{3}$, $\cos\gamma = \frac{1}{\sqrt{2}}$.

Find the value of $\tan(\alpha+\beta+\gamma)$ in surd form.

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(2) Mr. Ssentongo Leornard – 0782318244/701503577

19. Solve for x in the range $0°$ to $360°$
   (a) $3\cos^2 x - 3\sin x\cos x + 2\sin^2 x = 1$
   (b) $4\cos x = 3\tan x + 3\sec x$

20. Prove that $4\cos\theta\cos 3\theta + 1 = \dfrac{\sin 5\theta}{\sin\theta}$. Hence find all the values of θ in the range $0°$ to $180°$ for which $\cos\theta\cos 3\theta = \dfrac{-1}{2}$

VECTORS

21. The coordinates of the points A and B are (0,2,5) and (-1,3,1) and the equation of the line L is $\dfrac{x-3}{2} = \dfrac{y-2}{-2} = \dfrac{z-2}{-1}$
   (i) Find the equation of the plane containing the point A and perpendicular to L and verify that B lies in the plane.
   (ii) Show that the point C in which L meets the plane is (1,4,3) and find the angle between CA and CB

22. (a) A body moves such that its position is given by $OP = (3sint)i + (3cost)j$ where O is the origin and t is the time. Prove that the velocity of the particle when at P is perpendicular to OP.
   (b) The lines L_1 and L_2 have Cartesian equations $\dfrac{x}{1} = \dfrac{y+2}{2} = \dfrac{z-5}{-1}$ and $\dfrac{x-1}{-1} = \dfrac{y+3}{-3} = \dfrac{z-6}{1}$. Show that L_1 and L_2 intersect and find the coordinates of the point of intersection.

23. (a) Find the acute angle between the lines whose equations are $\dfrac{x-2}{-4} = \dfrac{y-3}{3} = \dfrac{z+1}{-1}$ and $\dfrac{x-3}{2} = \dfrac{y-1}{6} = \dfrac{z+1}{-5}$.
   (b) The points A and B have coordinates (1,2,3) and (4,6,-2) respectively and the plane has equation $x + y - z = 24$. Determine the equation of the line AB, hence the angle this line makes with the plane.

24. (a) Find the perpendicular distance of the line $\dfrac{x-5}{1} = \dfrac{y-6}{2} = \dfrac{z-3}{4}$ from the point (-6,-4,-5).
   (b) Find the shortest distance between the two skew lines $\dfrac{x+1}{1} = \dfrac{y-2}{2} = \dfrac{z-3}{1}$ and $\dfrac{x}{2} = \dfrac{y+1}{1} = \dfrac{z-1}{3}$ respectively.
   (c) Find the perpendicular distance of the plane $2x - 14z + 5z = 10$ from the origin.

25. (a) Show that the line $\dfrac{x-2}{2} = \dfrac{y-2}{-1} = \dfrac{z-3}{3}$ is parallel to the plane $4x - y - 3z = 4$ and find the perpendicular distance from the line to the plane.
   (b) Find the Cartesian equation of the line of intersection of the two planes $2x - 3y - z = 1$ and $3x + 4y + 2z = 3$.

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(2) Mr. Ssentongo Leornard – 0782318244/701503577

26. (a) Find the Cartesian equation of the plane containing the point (1,3,1) and parallel to the vectors $\begin{pmatrix}1\\-1\\3\end{pmatrix}$ and $\begin{pmatrix}2\\1\\-3\end{pmatrix}$

(b) Find the Cartesian equation of the plane containing the points (1,2,-1), (2,1,2) and (3,-3,3).

27. Given the points A, B and C with coordinates (2,5,-1), (3,-4,2) and C(-1,2,1). Show that ABC is a triangle and find the area of the triangle ABC

28. (a) Find the angle between the parallel planes $3x + 2y - z = -4$ and $6x + 4y - 2z = 6$.

(b) Find the acute angle between the planes $2x + y + 3z = 5$ and $2x + 3y + z = 7$

29. The points A and B have coordinates (2,1,1) and (0,5,3) respectively. Find the equation of the line AB. If C is the point (5,-4,2). Find the coordinates of D on AB such that CD is perpendicular to AB. Find the equation of the plane containing AB and perpendicular to the line CD.

30. (a) Given that $OP = \begin{pmatrix}4\\-3\\5\end{pmatrix}$ and $OQ = \begin{pmatrix}1\\0\\2\end{pmatrix}$, find the coordinates of the point R such that $\overline{PR} = \overline{PQ} = 1:2$ and the points P, Q and R are collinear.

(b) A and B are the points (3,1,1) and (5,2,3) respectively, and C is a point on the line $\boldsymbol{r} = \begin{pmatrix}2\\4\\-2\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$. If angle BAC=$90^0$, find the coordinates of C

ANALYSIS

31. Differentiate from first principles

(a) $y = \tan^{-1} x$

(b) $y = ax^n$

(c) $y = \sin 3x$

32. Find the derivative of:

(a) $y = 5\sin^{-1}(4x)$

(b) $y = \tan^{-1}\left(\dfrac{1+\tan x}{1-\tan x}\right)$

(c) $y = \dfrac{\sin x}{x^2 + \cos x}$

(d) $y = \sqrt{\dfrac{x}{1+x}}$

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(2) Mr. Ssentongo Leornard – 0782318244/701503577

33. Find:

(a) $\int \sin^{-1} x$

(b) $\int \frac{dx}{x^2+4x+13}$

(c) $\int \frac{dx}{x\log_e x}$

(d) $\int \frac{dx}{(1+x^2)\tan^{-1} x}$

(e) Show that $\int_0^2 \sqrt{\frac{x}{4-x}}dx = \pi - 2$

(f) Show that $\int_1^{10} x\log_{10} x = 50 - \frac{99}{4\ln 10}$

34. (a) If $x = t^3$ and $y = 2t^2$. Find $\frac{dy}{dx}$ in terms of t and show that when $\frac{dy}{dx} = 1$, $x = 2$ or $x = \frac{10}{27}$

(b) If $y = \frac{2t}{1+t^2}$ and $x = \frac{1-t^2}{1+t^2}$, find $\frac{d^2y}{dx^2}$ in terms of t

35. Given that:

(a) $y = \sqrt{4+3\sin x}$, show that $2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + y^2 = 4$

(b) $y = e^{2x}\cos 3x$, show that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$

(c) $y = \left(x+\sqrt{1+x^2}\right)^p$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - p^2y = 0$

(d) $y = \sin(\log_e x)$, show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

36. (a) Find the volume generated when the area enclosed by the curve $y = 4x - x^2$ and the line $y = 2x$ is rotated completely about the x – axis.

(b) Find the area contained between the two parabolas $4y = x^2$ and $4x = y^2$.

(c) Find the area between the curve $y = x^3$, the x – axis and the lines $y = 1, y = 8$.

(d) Find the area of the curve $x^2 + 3xy + 3y^2 = 1$

(e) Show that in the solid generated by the revolution of the rectangular hyperbola $x^2 - y^2 = a^2$ about the x – axis, the volume of the segment of height a from the vertex is $\frac{4}{3}\pi a^3$

37. (a) A right circular cone of semi – vertical angle θ is circumscribed about a sphere of radius R. show that the volume of the cone is $V = \frac{1}{3}\pi R^3(1+\operatorname{cosec}\theta)^3\tan^2\theta$ and find the value of θ when the volume is minimum.

Prepared by: (1) Mr. Mpagi Henry - 0774014461/758300086
(2) Mr. Ssentongo Leornard – 0782318244/701503577

(b) Water is poured into a vessel, in the shape of a right circular cone of vertical angle 90°, with the axis vertical, at the rate of $125\text{cm}^3/s$. At what rate is the water surface rising when the depth of the water is 10cm?

38. Sketch the curve $y = \frac{x}{x+2}$. Find the area enclosed by the curve, the lines $x = 0, x = 1$ and the line $y = 1$. Also find the volume generated when this area revolves through 2π radians about the line $y = 1$.

39. Solve the differential equations below:

(a) $\frac{1}{3x}\frac{dy}{dx} + \cos^2 y = 1$, when $x = 2$ and $y = \frac{\pi}{4}$

(b) $(x-y)\frac{dy}{dx} = x + y$, when $x = 4$ and $y = \pi$

(c) $\frac{dy}{dx} + 3y = e^{2x}$, when $x = 0$ and $y = \frac{6}{5}$

40. In a certain type of chemical reaction a substance A is continuously transformed into a substance B. throughout the reaction, the sum of the masses of A and B remains constant and equal to m. The mass of B present at time t after the commencement of the reaction is denoted by x. At any instant, the rate of increase of mass of B is k times the mass of A where k is a positive constant.

(a) Write down a differential equation relating x and t

(b) Solve this differential equation given that $x = 0$ and $t = 0$. Given also that $x = \frac{1}{2}m$ when $t = \ln 2$, determine the value of k and show that at time t, $x = m(1 - e^{-t})$. Hence find:

(i) The value of x (in terms of m) when $t = 3\ln 2$

(ii) The value of t when $x = \frac{3}{4}m$

GEOMETRY

41. (a) Find the equation of a line which makes an angle of 150° with the x – axis and y – intercept of -3 units.

(b) Find the acute angle between the lines $3y - x = 4$ and $6y - 3x - 5 = 0$

(c) OA and OB are equal sides of an isosceles triangle lying in the first quadrant. OA and OB make angles θ_1 and θ_2 with x – axis respectively. Show that the gradient of the bisector of the acute angle AOB is $\operatorname{cosec}\theta - \cot\theta$ where $\theta = \theta_1 + \theta_2$

(d) Find the length of the perpendicular from the point P(2,-4) to the line $3x + 2y - 5 = 0$

42. (a) Find the equation of the circle with centre (4,-7) which touches the line $3x + 4y - 9 = 0$

(b) Find the equation of the circle through the points (6,1), (3,2), (2,3)

(c) Find the equation of the circumcircle of the triangle formed by three lines $2y - 9x + 26 = 0$, $9y + 2x + 32 = 0$ and $11y - 7x - 27 = 0$

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(2) Mr. Ssentongo Leornard – 0782318244/701503577

43. (a) Find the length of the tangent from the point (5,6) to the circle $x^2+y^2+2x+4y-21=0$.

(b) Find the equations of the tangents to the circle $x^2+y^2=289$ which are parallel to the line $8x-15y=0$

(c) Find the equation of the circle of radius $12\tfrac{4}{5}$ which touches both the lines $4x-3y=0$ and $3x+4y-13=0$ and intersects the positive y – axis.

(d) A circle touches both the x – axis and the line $4x-3y+4=0$. Its centre is in the first quadrant and lies on the line $x-y-1=0$. Prove that its equation is $x^2+y^2-6x-4y+9=0$

44. Find the equations of the parabolas with the following foci and directrices:

(i) Focus (2,1), directrix $x=-3$

(ii) Focus (0,0), directrix $x+y=4$

(iii) Focus (-2,-3), directrix $3x+4y-3=0$

45. (a) Show that the curve $x=5-6y+y^2$ represents a parabola. Find its focus and directrix, hence sketch it.

(b) Find the equation of the normal to the curve $y^2=4bx$ at the point $P(bp^2,2bp)$. Given that the normal meets the curve again at $Q(bq^2,2bq)$, prove that $p^2+pq+2=0$

46. (a) Show that the equation of the normal with gradient m to the parabola $y^2=4ax$ is given by $y=mx-2am-am^3$.

(b) P and Q are two points on the parabola $y^2=4ax$ whose coordinates are $P(ap^2,2ap)$ and $Q(aq^2,2aq)$ respectively. If OP is perpendicular to OQ, show that $pq=-4$ and that the tangents to the curve at P and Q meet on the line $x+4a=0$

47. (a) A conic is given by $x=4\cos\theta$, $y=3\sin\theta$. Show that the conic is an ellipse and determine its eccentricity

(b) Given that the line $y=mx+c$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that $c^2=a^2m^2+b^2$. Hence determine the equations of the tangents at the point (-3,3) to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

48. (a) Show that the locus of the point of intersection of the tangents to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which are at right angles to one another is a circle $x^2+y^2=a^2+b^2$.

(b) The normal to the ellipse $x^2+y^2=100$ at the points A(6,4) and B(8,3) meet at N. If P is the mid – point of AB and O is the origin, show that OP is perpendicular to ON.

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(2) Mr. Ssentongo Leornard – 0782318244/701503577

49. (a) P is a point $\left(ap^2,2ap\right)$ and Q the point $\left(aq^2,2aq\right)$ on the parabola $y^2=4ax$. The tangents at P and Q intersect at R. Show that the area of triangle PQR is $\frac{1}{2}a^2\left(p-q\right)^3$

(b) The normal to the parabola $y^2=4ax$ at $P\left(ap^2,2ap\right)$ meets the axis of the parabola at M and MP is produced beyond P to Q so that MP = PQ. Show that the locus of Q is $y^2=16a\left(x+2a\right)$

50. (a) The normal to the rectangular hyperbola $xy=8$ at the point (4,2) meets the asymptotes at M and N. Find the length of MN

(b) The tangent at P to the rectangular hyperbola $xy=c^2$ meets the lines $x-y=0$ and $x+y=0$ at A and B and Δ denotes the area of triangle OAB where O is the origin. The normal at P meets the x – axis at C and the y – axis at D. if Δ_1 denotes the area of the triangle ODC. Show that $\Delta^2\Delta_1=8c^6$

END

Prepared by: (1) Mr. Mpagi Henry - 0774014461/758300086
(2) Mr. Ssentongo Leornard – 0782318244/701503577

Permutations and combinations

Permutation

A permutation is an ordered arrangement of a number of objects

Consider digits 1, 2 and 3; find the possible arrangements of the digits

$$\left.\begin{matrix}123, 132, 321\\ 231, 321, 321\end{matrix}\right\} \text{The total of six}$$

This problem may also be solved as follows:

Given the three digits above, the first position can take up three digits, the second position can take up two digits and the third position can take up 1 digit only

1st position	2nd position	3rd position
3	2	1

The total is thus 3 x 2 x 1 = 6

If the digits were four say 1, 2, 3, 4 the arrangement would be

1st	2nd	3rd	4th
4	3	2	1

The total is thus 4 x 3 x 2 x 1 = 24

In summary the number of ways of arranging n different items in a row is given by n(n – 1)(n – 2)(n -3) x ……….2 x 1 and can be expressed as n!

If the total number of books is 6

The total number of arrangements = 6!

= 6 x 5 x 4 x 3 x 2 x 1 = 720 ways

Example 1

Find the values of the following expression

(a) 5!
Solution
5!= 5 x 4 x 3 x 2 x 1 = 120

(b) $\frac{8!}{5!}$
Solution
$\frac{8!}{5!} = \frac{8\,x\,7\,x\,6\,x\,5!}{5!} = 336$

(c) $\frac{10!}{6!x\,5!\,x\,2!}$
Solution
$\frac{10!}{6!x\,5!\,x\,2!} = \frac{10\,x\,9\,x\,8\,x\,7\,x\,6!}{6!x\,5\,x\,4\,x\,3\,x\,2\,x\,1\,x\,2x\,1} = 21$

(d) Four different pens and 5 different books are to be arranged on a row. Find

(i) The number of possible arrangements of items
Solution
Total number of items = 4 + 5 = 9
Total number of arrangements = 9!
= 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
= 362,880 ways

(ii) The number of possible arrangements if three of books must be kept together
Solution
The pens are taken to be one since they are to be kept together. So we consider total number of items to six.
The number of arrangements of six items = 6! = 6 x 5 x 4 x 3 x 2 x 1
= 720 ways
The arrangement of 4 pens = 4!
= 4 x 3 x 2 x 1 =24
Total number arrangements of all the items = 720 x 24 = 17, 280

Multiplication principle of permutation

If one operation can be performed independently in a different ways and the second in b different ways, then either of the two events can be performed in (a + b) ways

Example 2

There are 6 roads joining P to Q and 3 roads joining Q to R. Find how many possible routes are from P to R

From P to Q = 6 ways

From Q to R = 3 ways

Number of routes from P to R = 6 x 3 = 18

Example 3

Peter can eat either matooke, rice or posh on any of the seven days of the week. In how many ways can he arrange his meals in a week

Solution

For each of the 7 days, there are 3 choices

Total number of arrangements

= 3 x 3 x 3 x 3 x 3 x 3 x 3 = 3^7 = 2187 ways

Example 4

There are four routes from Nairobi to Mombasa. In how many different ways can a taxi go from Nairobi to Mombasa and returning if for returning:

(a) any of the route is taken
= 4 x 4 = 16 ways
(b) the same route is taken
= 4 x 1 = 4 ways
(c) the same route is not taken
= 4 x 3 = 12 ways

Example 5

David can arrange a set of items in 5 ways and John can arrange the same set of items in 3 ways. In how many ways can either David or John arrange the items?

Solution

Number of ways in which David arranges = 5

Number of ways in which John arranges = 3

Number of ways in which either David or John arrange the items = 5 + 3 = 8 ways

The number of permutation of r objects taken from n unlike objects

The permutation of n unlike objects taking r at a time is denoted by ^nP_r which is defined as

$^nP_r = \frac{n!}{(n-r)!}$, where r ≤ n.

In case r = n, we have ^nP_n which is interpreted as the number of arranging n chosen objects from n objects denoted by n!

$^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n! => 0! = 1$

Example 6

How many three letter words can be formed from the sample space {a, b, c, d, e, f}

Solution

Total number of letters = 6 and r = 3

Total number of worms = $^6P_3$

$= \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6\ x\ 5\ x\ 4\ x\ 3!}{3!} = 120$ways

Example 7

Find the possible number of ways of arranging 3 letters from the word MANGOES

Solution

Total number of letter in the word = 7

and r = 3

Number of ways $^7P^3 = \frac{7!}{(7-3)!}$

$=\frac{7\ x\ 6\ x\ 5\ x\ 4\ x\ 3!}{3!} = 840$ways

Example 8

Find number of ways of arranging six boys from a group of 13

Solution

Number of arrangements = $^{13}P_6$

$=\frac{13!}{(13-6)!} = \frac{13!}{7!}$

$= \frac{13x12x11x10\ x9\ x\ 8\ x\ 7!}{7!}$

= 1235520 ways

The number of permutations of n objects of which r are alike

The number of permutations of n objects of which r are alike is given by $\frac{n!}{r!}$

Example 9

Find the number of arranging in a line the letters B, C, C, C, C, C, C

The number of ways of arranging the seven letters of which of which 6 are alike
$= \frac{7!}{6!} = \frac{7\ x\ 6!}{6!} = 7$ ways

The number of ways of permutations on n objects of which p of one type are alike, q of the second type are alike, r of the third type are alike, and so on.

The number of ways of permutations on n objects of which p of one type are alike, q of the second type are alike, r of the third type are alike given by $\frac{n!}{p!\ x\ q!\ x\ r!}$

Example 10

Find the possible number of ways of arranging the letter of the word MATHEMATICS in line

Solution

The word MATHEMATICS has 11 letters and contains 2 M, 2A and 2T repeated

The number of ways = $\frac{11!}{2!x\ 2!\ x\ 2!}$

= $\frac{11\ x\ 10\ x\ 9\ \ x\ 8\ x\ 7\ x\ 6\ x\ 5\ x\ 4x3\ x\ 2\ x\ 1}{2\ x\ 2\ x\ 2}$

= 4,989,600

Example 11

Find the possible number of ways of arranging the letter of the word 'MISSISSIPPI' in line

Solution

The word 'MISSISSIPPI' has 10 letters with 4I, 4S, and 2P

The number of ways = $\frac{11!}{4!x\ 4!\ x\ 2!} = 34650$

The number of permutations of the like and unlike objects with restrictions

One should be cautious when handling these problems

Example 12

Find the possible number of ways of arranging

The letters of the word MINIMUM if the arrangement begins with MMM?

Solution

There is only one way of arranging MMM

The remaining contain four letters with 2I can be arranged in

$\frac{1\ x\ 4!}{2!} = \frac{1\ x\ 4\ x\ 3\ x\ 2!}{2!} = 12$ ways

Example 13

(a) How many 4 digit number greater than 6000 can be formed from 4, 5, 6, 7, 8 and 9 if:

(i) Repetitions are allowed

Solution

The first digit can be chosen from 6, 7, 8 and 9, hence 4 possible ways, the 2nd, 3rd and 4th are chosen from any of the six digits since repetitions are allowed

position	1st	2nd	3rd	4th
selections	4	6	6	6

Number of ways

= 4 x 6 x 6 x 6 = 864 ways

(ii) Repetition are not allowed

The first can be chosen from 6, 7, 8 and 9, hence 4 possible ways, the 2nd from 5, 3rd from 4 and 4th from 3 since no repetitions are allowed

position	1st	2nd	3rd	4th
selections	4	5	4	3

Number of ways
= 4 x 5 x 4 x 3 = 240 ways

(b) Find how many four digit numbers can be formed from the six digits 2, 3, 5, 7, 8 and 9 without repeating any digit.
Find also how many of these numbers
(i) Are less than 7000
(ii) Are odd

Solution

position	1^{st}	2^{nd}	3^{rd}	4^{th}
selections	6	5	4	3

Total number of ways = 6 x 5 x 4 x 3 = 360

(i) The 1^{st} number is selected from three (2, 3, 5), the 2^{nd} number from 5, the 3^{rd} from 4 and the 4^{th} from 3 digits

position	1^{st}	2^{nd}	3^{rd}	4^{th}
selections	6	5	4	3

Total number of less than 7000

= 3 x 5 x 4 x 3 = 180

(ii) The last number is selected from four odd digits (3, 5, 7, and 9), the 1^{st} number selected from five remaining, 2^{nd} from 4 and 3^{rd} from 3

position	1^{st}	2^{nd}	3^{rd}	4^{th}
selections	5	4	3	4

Total number of odd numbers formed

= 5 x 4 x 3 x 4 = 240

(c) How many different 6 digit number greater than 500000 can be formed by using the digits 1, 5, 7, 7, 7, 8
Solution
The 1^{st} digit is selected from five (5, 7, 7, 7, 8), the 2^{nd} from remaining five, 3^{rd} from four, 4^{th} from three, 5^{th} from two and 6^{th} from one

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6th
5	5	4	3	2	1

Total number = $\frac{5\ x\ 5\ x\ 4\ x\ 3\ x\ 2\ x\ 1}{3!}$ = 100
NB. The number is divided by 3! Because 7 appears three times

(d) How many odd numbers greater than 60000 can be formed from 0, 5, 6, 7, 8, 9, if no number contains any digit more than once
Solution
Considering six digits
Taking the first digit to be odd, the first digit is selected from 3 digits (5, 7, 9) and the last is selected from 2 digits

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6th
3	4	3	2	1	2

Number of ways = 3 x 4 x 3 x 2 x 1 x 2
= 144
Taking the first digit to be even, the first digit is selected from 2 digits (6, 8) since the number should be greater than 60000 and the last is selected from 2 odd digits

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6th
2	4	3	2	1	3

Number of ways = 3 x 4 x 3 x 2 x 1 x 2
= 144
Considering five digits
Taking the first digit to be odd, the digit greater than 6 are 7 and 9 so first digit is selected from 2 digits and the last is selected from 2 digits

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
2	4	3	2	2

Number of ways = 2 x 4 x 3 x 2 x 2
= 96
Taking the first digit to be even, the first digit is selected from 2 digits (6, 8) since the number should be greater than 60000 and the last is selected from 3 odd digits (5, 7, 9)

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
2	4	3	2	3

Number of ways = 2 x 4 x 3 x 2 x 3
= 144
The total number of selections
= 144 + 144 + 96 + 144 = 528

Example 14

The six letter of the word LONDON are each written on a card and the six cards are shuffled and placed in a line. Find the number of possible arrangements if

(a) The middle two cards both have the letter N on them

Solution

If the middle letter are NN, the we need to find the number of different arrangements of the letter LODO. With the 2O's, the number of arrangements = $\frac{4!}{2!} = 12$

(b) The two cards with letter O are not adjacent and the two cards with letter N are also not adjacent

Solution

If the two cards are not adjacent, the number of arrangements = Total number of arrangements of the word LONDON – number of arrangements when the two letters are adjacent

= = $\frac{6!}{2!2!} - 24 = 156$

Example 15

In how many different ways can letters of the word MISCHIEVERS be arranged if the S's cannot be together

Solution

There are 11 letters in the word MISCHIEVERS with 2S's, 2I's and 2E's

Total number of arrangements

= $\frac{11!}{2!2!2!}$ = 4989600

If S's are together, we consider them as one, so the number of arrangements

= $\frac{10!}{2!2!}$ = 907200

∴ the number of possible arrangements of the word MISCHIEVERS when S's are not together

= 4989600 – 907200 = 4082400

The number of permutation of n different objects taken r at a time, if repetition are permitted

Example 16

How many four digit numbers can be formed from the sample space {1, 2, 3, 4, 5} if repetitions are permissible

Solution

The 1st position has five possibilities, the 2nd five, the 3rd five, the 4th five

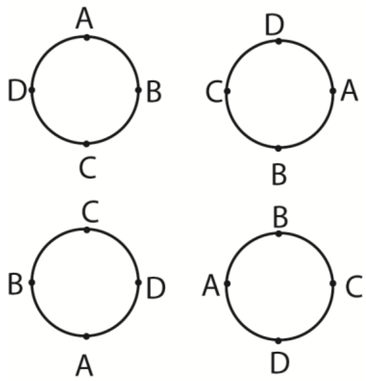
Number of permutations = 5 x 5 x 5 x 5 = 625

Circular permutations

Here objects are arrange in a circle

The number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise are different.

Consider four people A, B, C and D seated at a round table. The possible arrangements are as shown below



With circular arrangements of this type, it is the relative positions of the objects being arranged which is important. The arrangements of the people above is the same. However, if the people were seated in a line the arrangements would not be the same, i.e. A, B, C, D is not the same as D, A, B, C. When finding the number of different arrangements, we fix one person say A and find the number of ways of arranging B, C and D.

Therefore, the number of different arrangements of four people around the table is 3!

Hence the number of different arrangements of n people seated around a table is (n – 1)!

Example 17

(a) Seven people are to be seated around a table, in how many ways can this be done
Solution
The number of ways = (7 – 1)! = 6!
= 720
(b) In how many ways can five people A, B, C, D and E be seated at a round table if
(i) A must be seated next to B

Solution

If A and B are seated together, they are taken as bound together. So four people are considered

The number of ways = (4 – 1)! = 3! = 6

The number of ways in which A and B can be arranged = 2

The total number of arrangements

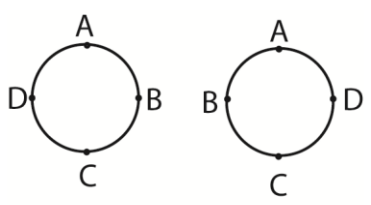
= 6 x 2 = 12 ways
(ii) A must not seat next to B

If A and B are not seated together, then the number of arrangements = total number of arrangements – number of arrangements when A and B are seated together

= (5 – 1)! – 12 = 12 ways

The number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise arrangements are the same

Consider the four people above, if the arrangement is as shown below



Then the above arrangements are the same since one is the other viewed from the opposite side

The number of arrangements = $\frac{3!}{2}$ = 3ways

Hence the number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise arrangements are the same = $\frac{(n-1)!}{2}$

Example 18

A white, a blue, a red and two yellow cards at arranged on a circle. Find the number of arrangements if red and white cards are next to each other.

Solution

If red and white cards are next to each other, they are considered as bound together. So we have four cards. Since anticlockwise and clockwise arrangements are the same and there are two yellow cards, the number of arrangements = $\frac{(4-1)!}{2\ x\ 2!} = \frac{3!}{2\ x\ 2!}$

The number of ways of arranging red and white cards = 2

Total number of ways of arrangements

=$\frac{3!}{2\ x\ 2!}\ x$ 2 = 3

Revision exercise 1

1. In how many ways can the letters of the words below be arranged
   (a) Bbosa (5!]
   (b) Precious [8!]
2. How many different arrangements of the letters of the word PARALLELOGRAM can be made with A's separate [83160000]
3. How many different arrangements of the letters of the word CONTACT can be made with vowels separated? [900]
4. How many odd numbers greater than 6000 can be formed using digits 2, 3, 4, 5 and 6 if each digit is used only once in each number [12]

5. Three boys and five girls are to be seated on a bench such that the eldest girl and eldest boy sit next to each other. In how many ways can this be done [2 x 7!]
6. A round table conference is to be held between delegates of 12 countries. In how many ways can they be seated if two particular delegates wish to sit together [2 x 10!]
7. In how many ways can 4 boys and 4 girls be seated at a circular table such that no two boys are adjacent [144]
8. How many words beginning or ending with a consonant can be formed by using the letters of the word EQUATION? [4320]

Combinations

A combination is a selection of items from a group not basing on the order in which the items are selected

Consider the letters A, B, C, D

The possible arrangements of two letters chosen from the above letters are

AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC. AS seen earlier, the total number of arrangements of the above letters is expressed as $\frac{4!}{(4-2)!} = \frac{4!}{2!} = 12$

However, when considering combinations, the grouping such as AB and BA are said to be the same groupings such as CA and AC, AD and DA, etc.

So the possible combinations are AB, AC, AD, BC, BD, CD which is six ways.

Thus the number of possible combinations of n items taken r at a time is expressed as ^nC_r or $\binom{n}{r}$ which is defined as $^nC_r = \frac{n!}{(n-r)!r!}$ where $r \leq n$

Hence the number of combinations of the above letters taken two at a time is

$\binom{4}{2} = \frac{4!}{2!2!} = 6$

Example 19

A committee of four people is chosen at random from a set of seven men and three women

How many different groups can be chosen if there is at least one

(i) Woman on the committee

Solution

Possible combinations

7 men	3 women
3	1
2	2
1	3

The number of ways of choosing at least one woman

$=\binom{7}{3} x \binom{3}{1} + \binom{7}{2} x \binom{3}{2} + \binom{7}{1} x \binom{3}{3}$

$=\frac{7!}{3!4!} x \frac{3!}{1!2!} + \frac{7!}{2!5!} x \frac{3!}{2!1!} + \frac{7!}{1!6!} x \frac{3!}{3!0!} = 175$

(ii) Man on the committee

7 men	3 women
1	3
2	2
3	1
4	0

The number of ways of choosing at least one man

$\binom{7}{1} x \binom{3}{3} + \binom{7}{2} x \binom{3}{2} + \binom{7}{3} x \binom{3}{1} + \binom{7}{4} x \binom{3}{0}$

$=\frac{7!}{1!3!} x \frac{3!}{0!3!} + \frac{7!}{5!2!} x \frac{3!}{1!2!} + \frac{7!}{4!3!} x \frac{3!}{2!1!} + \frac{7!}{3!4!} x \frac{3!}{3!0!}$

=210

Example 20

A group of nine has to be selected from ten men and eight women. It can consist of either five men and four women or four men and five women. How many different groups can be chosen?

Solution

Possible combination

10 men	8 women
5	4
4	5

Number of groups = $\binom{10}{5}x\binom{8}{4}+\binom{10}{4}x\binom{8}{5}$

= $\frac{10!}{5!5!}$ x $\frac{8!}{4!4!}$ + $\frac{10!}{6!4!}$ x $\frac{8!}{5!3!}$ = 29400

Example 21

A team of six is to be formed from 13 boys and 7 girls. In how many ways can the team be selected if it must consist of

(a) 4 boy and 2 girls

13 boys	7 girls
4	2

$\binom{13}{4}.\binom{7}{2}$= $\frac{13!}{9!4!}x\frac{7!}{5!2!}$ = 15015

(b) At least one member of each sex

Possible combinations

13 boys	7 girls
5	1
4	2
3	3
2	4
1	5

$=\binom{13}{5}.\binom{7}{1}+\binom{13}{4}.\binom{7}{2}+\binom{13}{3}.\binom{7}{3}+\binom{13}{2}.\binom{7}{4}+\binom{13}{1}.\binom{7}{5}$

$=\frac{13!}{8!5!}.\frac{7!}{6!1!}+\frac{13!}{9!4!}.\frac{7!}{5!2!}+\frac{13!}{10!3!}.\frac{7!}{4!3!}+\frac{13!}{11!2!}.\frac{7!}{3!4!}+\frac{13!}{12!1!}.\frac{7!}{2!5!}$

= 37037

Example 22

A team of 11 players is to be chosen from a group of 15 players. Two of the 11 are to be randomly elected a captain and vice-captain respectively. In how many ways can this be done?

Number of ways of choosing 11 players from 15 = $\binom{15}{11}$

A captain will be elected from 11 players and a vice-captain from 10 players

Total number of selection =$\binom{15}{11}\ x\ 11\ x\ 10$

=$\frac{15!}{4!11!}$ x 11 x 10 = 150150

Example 23

(a) Find the number of different selections of 4 letters that can be made from the word UNDERMATCH.
Solution
There are 10 letters which are all different
Number of selections of 4 letters from 10 is given by $\binom{10}{4}=\frac{10!}{(10-4)!4!}=\frac{10!}{6!4!}=210$

(b) How many selections do not contain a vowel?
Solution
Number of vowels in the word = 2
Number of letters not vowels = 8
Number of selections of 4 letters from 10 without containing a vowel = selecting 4 letters from 8 consonants =
$\binom{8}{4}=\frac{8!}{(8-4)!4!}=\frac{8!}{4!4!}=70$

Example 24

In how many ways can three letters be selected at random from the word BIOLOGY is selection

(a) Does not contain the letter O
Solution
Number of selections without the letter O = number of ways of choosing three letters from B, I, L, G, y
$=\binom{5}{3}=\frac{5!}{2!3!}=10$

(b) Contain only the letter O

Solution

Number of selections with one letter O = number of ways of choosing two letters from B, I, L, G, y

$=\binom{5}{2}=\frac{5!}{3!2!}=10$

(c) Contains both of the letters O

Solution

Number of selections with two letter O = number of ways of choosing one letter from B, I, L, G, y

$=\binom{5}{1}=\frac{5!}{4!1!}=5$

Example 25

In how many ways can four letters be selected at random from the word BREAKDOWN if the letters contain at least one vowel?

Solution

Vowels: E, A, O (3)

Consonants: B, R, K, D, W, N (6)

Consonants (6)	Vowels (3)
3	1
2	2
1	3

Number of selection of four letters with at least one vowel

$$= \binom{6}{3}.\binom{3}{1} + \binom{6}{2}.\binom{3}{2} + \binom{6}{1}.\binom{3}{3} = 111$$

Example 26

How many different selections can be made from the six digits 1, 2, 3, 4, 5, 6

Solution

Note: this an open questions because selections can consist of only one digit, two digits, three digits, four digits, five digits or six digits

Number of selection of 1 digit = $^6C_1$ = 6

Number of selection of 2 digits = $^6C_2$ = 15

Number of selection of 3 digits = $^6C_3$ = 20

Number of selection of 4 digits = $^6C_4$ = 15

Number of selection of 5 digits = $^6C_5$ = 6

Number of selection of 6 digits = $^6C_1$ = 1

Total number of selections

= 6 + 15 + 20 + 15 +6 + 1 = 63

This approach is tedious for a large group of objects.

The general formula for selection from n unlike objects is given by $2^n - 1$.

For the above problems, number of selections = $2^6 - 1$ = 63

Example 26

How many different selections can be made from 26 different letters of the alphabet?

Number of selection = $2^{26} - 1$

= 67,108,863

Cases involving repetitions

Suppose we need to find the number of possible selections of letters from a word containing repeated letters, we take the selections mutually exclusive

Example 27

How many different selections can be made from the letters of the word CANADIAN?

Solution

There are 3A's, 2N's and 3 other letters

The A's can be dealt with in 4 ways (either no A, 1A's, 2A's or 3A's)

The N's can be dealt in 3 ways (no N, 1N, or 2N's)

The C can be dealt with in 2 ways (no C, 1C)

The D can be dealt with in 2 ways (no D, 1D)

The I can be dealt with in 2 ways (no I, 1I)

The number of selections

= 4 x 3 x 2 x 2 x 2 – 1 = 95

Example 28

How many different selections can be made from the letters of the word POSSESS?

Solution

There are 4S's and 3 other letters

The S's can be dealt in 5 ways (no S, 1S, 2N's, 3S's, 4S's, or 5S's)

The P can be dealt with in 2 ways (no P, 1P)

The O can be dealt with in 2 ways (no O, 1O)

The E can be dealt with in 2 ways (no E, 1E)

Total number of selections = 5 x 2 x 2 x 2 – 1

= 39

Cases involving division into groups

The number of ways of dividing n unlike objects into say two groups of p and q where p + q = n is given by $\frac{n!}{p!q!}$

For three groups of p, q and r provided p + q + r = n

Number of ways of division = $\frac{n!}{p!q!r!}$

However, for the two groups above, if p = q then the number of ways of division = $\frac{n!}{p!p!2!}$

For three groups where p = q = r

then the number of ways of division = $\frac{n!}{p!p!p!3!}$

Example 29

The following letters a, b, c, d, e, f, g, h, I, j, k, l are to be divided into groups containing

(a) 3, 4, 5
(b) 5, 7
(c) 6, 6
(d) 4, 4, 4 letters. In how many ways can this be done?

Solution

(a) Number of ways = $\frac{12!}{3!4!5!}$ = 27720
(b) Number of ways = $\frac{12!}{5!7!}$ = 792
(c) Number of ways = $\frac{12!}{6!6!2!}$ = 462
(d) Number of ways = $\frac{12!}{4!4!4!3!}$ =5775

Example 30

Find the number of ways that 18 objects can be arranged into groups if there are to be

(a) Two groups of 9 objects each
(b) Three groups of 6 objects each
(c) 6 groups of 3 objects each
(d) Three groups of 5, 6 and 7 objects each

Solution

(a) Number of ways = $\frac{18!}{9!9!2!}$ = 24310
(b) Number of ways = $\frac{18!}{6!6!6!3!}$ = 2858856
(c) Number of ways = $\frac{18!}{3!3!3!3!3!3!6!}$ =190590400
(d) Number of ways = $\frac{18!}{5!6!7!}$ = 14702688

Example 31

(a) Find how many words can be formed using all letters in the word MINIMUM.

Solution

Number of ways of arranging the letters = 7!

There are 3M's and 2I's

Number of words formed = $\frac{7!}{3!2!} = 420$

(b) Compute the sum of four-digit numbers formed with the four digits 2, 5, 3, 8 if each digit is used only once in each arrangement

Solution

Number of ways of arranging a four digit number = 4!

Sum of any four digit number formed
= 2 + 5 + 3 + 8 = 18

Total sum of four digit numbers formed

= 18 x 4! = 432

(c) A committee consisting of 2 men and 3 women is to be formed from a group of 5 men and 7 women. Find the number of different committees that can be formed. If two of the women refuse to serve on the same committee, how many committees can be formed?

Solution

The committees formed = ${}^5C_2.{}^7C_3$

= 10 x 35 = 350

Suppose two women are to serve together, we take them as glued together, so the number of committees =${}^5C_2.{}^6C_3$ = 200

Number of committees in which two women refuse to serve together = 350 – 200 = 150

Revision exercise 2

1. (a) Find the number of different selection of 3 letters that can be made from the word PHOTOGRAPH. [53]
   (b) How many of these selections contain no vowel [18]
   (c) How many of these selections contain at least one vowel? [35]
2. (a) find the number of different selections of 3 letters that can be made from the letters of the word SUCCESSFUL.[36]
   (c) How many of these selections contain only consonants [11]
   (d) How many of these selections contain at least one vowel [25]
3. (a) Find the value of n if ${}^{n}P_4 = 30{}^{n}C_5$ [8]
   (b) How many arrangement can be made from the letters of the name MISSISSIPPI
      (i) when all the letters are taken at a time [34650]

      (ii) If the two letters PP begin every word [630 ways]

   (c) Find the number of ways in which a one can chose one or more of the four girls to join a discussion group [15 ways]
4. Find in how many ways 11 people can be divided into three groups containing 3, 4, 4 people each. [5775]
5. A group of 5 boys and 8 girls. In how many ways can a team of four be chosen, if the team contains
   (a) No girl [5]
   (b) No more than one girl [85]
   (c) At least two boys [365]
6. Calculate the number of 7 – letter arrangements which can be made with the letters of the word MAXIMUM. In how many of these do all the 4 consonants appear next to each other? [840, 96]
7. In how many ways can a club of 5 be selected from 7 boys and 3 girls if it must contain
   (a) 3 boys and 3 girls [105]
   (b) 2 men and 3 girls [21]
   (c) At least one girl [231]
8. How many different 6 digit numbers greater than 400,000 can be formed form the following digits 1, 4, 6, 6, 6 7? [100]

Thank you

Dr. Bbosa Science

SENIOR SIX MATHEMATICS SEMINAR TO BE HELD ON 27TH FEBRUARY 2021 ONLINE SEMINAR THROUGH ZOOM.

APPLIED MATHEMATICS P425/2

Instructions:

As a school, please select several questions and prepare presentations in each. We shall book the questions of choice through the WhatsApp group and on the seminar day.

STATISTICS/PROBABILITY:

1. Box A contains 4 red and 5 blue pens, while box B contains 4 red and 4 blue pens. It is known that box T is three times likely to be selected as box Y. A box is selected at random and two pens are drawn from it without replacement.

   (a) Find the probability of picking pens of different colours.

   (b) Construct a probability distribution for the number of blue pens that are drawn. Hence calculate the variance of the number of pens drawn from the box.

2. The following table gives the distribution of the interest paid to 500 shareholders of Moringa Growers' Association at the end of 2000.

Interest x 1000 USh	25 -	30 -	40 -	60 -	80 -	110 -	120- < 130
No. of share holders	17	55	142	153	93	20	20

i) Draw a histogram to illustrate this data.

ii) Find the average interest each shareholder receives.

(iii) Calculate the standard deviation of the distribution above.

(iv) Using a cumulative frequency curve or otherwise, find the range of the interest obtained by the middle 80% of the shareholders.

(b) A four-man team is to be selected from three women and 4 men,

(a) What is the probability that (i) the women will form the majority?

(ii) at least 3 men will be on the committee?

(b) If the group contains a couple that insists on being on the committee together repeat question (a)(i) and (ii) above.

3. In a machine manufacturing company, 85% of the nails made are approximately within the set tolerance limits. If a random sample of 200 nails is taken, find the probability that;

(i) exactly 33 are outside the tolerance limits.

(ii) between 21 and 27 nails, inclusive, will be outside the tolerance limits.

4. The continuous random variable X has a probability function,

$$f(x)=\begin{cases} k(x+2); & -1 < x < 0 \\ 2k; & 0 \leq x \leq 1 \\ \frac{k(5-x)}{2}; & 1 < x \leq 3 \\ 0, & elswhere \end{cases}$$

where k is a constant.

(a) Sketch f(x) and hence find the constant, k

(b) median

(c) $P(\frac{1}{2} < x < 2)$

5. (a) The weights of ball bearings are normally distributed with mean 25gram and standard deviation 4 grams. If a random sample of 16 ball bearings is taken, find the;

(i) Probability that the mean of the sample is between 24.12 grams and 26.73 grams.

(ii) Interquartile range.

(b) A random sample of 120 girls taken from a normally distributed population of girls school gave a mean age of 16.5 and variance of 18. Determine the 97% confidence interval for the mean age of all the school girls.

MECHANICS:

6. (a) A car traveling at 54 kmh^{-1} is brought to rest with uniform retardation in 5 seconds. Find its retardation in ms^{-2} and distance it travelled in this time

   (b) A cyclist was timed between successive trading centres P; Q and R, each 2 km a part. It took $\frac{5}{3}$ minutes to travel from P to Q and 2.5 minutes from Q to R. Find;

   (i) the acceleration

   (ii) the velocity with which the cyclist passes point P

   (iii) How much further the cyclist will travel before coming to rest if the acceleration remains uniform.

7. A bullet is fired from a point P which is at the top of a hill 50m above the ground.

The speed with which the bullet is fired is 140ms^{-1} and it hits the ground at a point Q which is at a horizontal distance 200m from the foot of the bill. Find the,

(i) two possible values of angle of projection

(ii) two possible times of flight.

(iii) angle with which the bullet hits the grounds.

8. (a) A body of mass 8kg in contact with a rough plane inclined at 50º to the horizontal is just prevented from sliding down the plane by a horizontal force P. if the angle of friction between the plane and the body is 25º. Calculate the magnitude of P.

   (b)Two smooth inclined planes meet at right angles, the inclination of one to the horizontal being 30° and the other being 60°. Bodies of masses 2kg and 4kg lie on the respective planes, and the two masses are joined by a light inextensible string passing over a smooth fixed pulley at the intersection of the planes.

   (i) If the masses are released from rest, find the acceleration of the masses and tension of the string.

   (ii) If the surface with 4kg mass experiences a frictional force of 0.25N, find the new acceleration of the masses when the system is set from rest.

9. (a) A pump draws water from a tank and issues it at a speed of 8 ms^{-1} from the end of a pipe of cross-sectional area of $0.01m^2$ situated 10m above the level from which the water is drawn. Find the rate at which the pump is working (take density of water = $1000kgm^{-3}$)

(b) A car of mass 800kg is pulling a trailer of mass 200kg up a hill inclined at an angle $Sin^{-1}\left(\frac{1}{14}\right)$ to the horizontal. When the total force exerted by the engine is 1000N the car and the trailer move up the hill at a steady speed.

(i) Find the total frictional resistance to the motion of the car and trailer during this motion

(ii) If the frictional resistance on the car is 280N, find the tension in the coupling between the car and the trailer.

10. A particle A initially at the point with position vector $2\mathbf{i} - 5\mathbf{j} + \mathbf{k}$ km is moving with a constant velocity of $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ kmh^{-1}. At the same instant, a particle B at the point $(3, 3, 2)$ is moving with a constant velocity of $3\mathbf{i} - 2\mathbf{k}$ kmh^{-1}. Find the:

(i) relative velocity of particle A to B.

(ii) relative displacement of particle A to particle B at any instant.

(iii) shortest distance between the two particles in their subsequent motion.

11. A body moving with an acceleration $(2e^{4t}\boldsymbol{i} - 3Cost\ \boldsymbol{j} + 4Sin2t\ \boldsymbol{k})$ m/s^2 is initially located at a point whose position vector is $(\boldsymbol{i} - \boldsymbol{2j} + \boldsymbol{k})m$ and has a velocity $(\boldsymbol{i} - \boldsymbol{2j} + \boldsymbol{2k})\ m/s$ at time t = 0.

Find the (a) velocity of the body at time t= $\frac{\pi}{2}$ s

(b) displacement of the body at any time t =1s.

12. A non-uniform ladder AB whose centre of gravity is 2m from end A is of length 6m and weight, **W**. The ladder is inclined at an angle θ to the vertical with its end B against a rough vertical wall and end A on a rough horizontal ground with which the coefficients of friction at each point of contact is μ. If the ladder is about to slip when a man of weight **5W** ascends two-thirds of the way up the ladder, show that $\boldsymbol{tan\theta = \frac{18\mu}{11-7\mu^2}}$

13. (a) ABCD is a square of side 2m. Forces of magnitudes 3N, 5N, 7N and 2N act a long sides DA, AB, BC and CD respectively. Calculate:

(i) the magnitude of the resultant of the forces and the angle made by the resultant with AD.

(ii) the sum of the moments of the forces about A

(iii) the distance from A of the point where the line of action of the resultant of the forces cuts DA produced.

(iv) the equation of the line of action of the resultant.

14. (a) A smooth bead of mass 0.2 kg is threaded on a smooth circular wire of radius r metres which is held in a vertical plane. If the bead is projected from the lowest point on the circle with speed $\sqrt{3rg}$. Find the;

(a) speed of the bead when it has gone one sixth of the way round the circle.

(b) force exerted on the bead by the wire at this point.

NUMERICAL METHODS:

15. (a) Give X = 12.6955

i) Truncate to 2 significant figures   ii) Round to 2 decimal places

b) Given A = 7.684, B = 0.31, Write down the maximum possible errors in A and B. Hence, find; (i) absolute error in the quotient $\frac{A+B}{AB}$

(ii) Interval within which the quotient in b (i) above lies.

c) The quantities **p** and **q** were measured with errors e_1 and $e_2$respectively. Show that the maximum relative error in $\frac{p}{\sqrt{q}}$ is given by $\left|\frac{e_1}{p}\right| + \frac{1}{2}\left|\frac{e_2}{q}\right|$

Hence find the range within which $\frac{2.35}{\sqrt{5.1}}$ lies

16. a) A school canteen started its operation with a capital of 2 million shillings. At the end of a certain term its profits in soft drinks and foodstuff were 0.2 million and 0.45 million respectively. There were possible errors of 6.5% and 8.2% in the sales respectively. Find the maximum and minimum values of the sales as a percentage of the capital.

b) The charges of sending parcels by a certain, distributing company depends on the weights, 500g, 1kg, 1.5kg, 2kg and 5kg the charges are 750/=, 1000/=, 2000/=. 3500/=, and 5500/= respectively. Estimate;

i) What would the distributor charge for a parcel of weight 2.7kg?

ii) If the sender pays 6200/=, what is the weight of the parcel?

17. (a) Show that the Newton – Raphson formula for solving the transcendental equation

$\sin x - \frac{1}{2} = 0$ is given by $x_{n+1} = \dfrac{x_n \cos x_n - \sin x_n + \dfrac{1}{2}}{\cos x_n}$

Show also that one of the roots of the equation lies between 1.5 and 2. Hence find the root to two decimal places.

18. (a) Use the trapezium rule with 6 ordinates to find $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos x}\,dx$ correct to three decimal places.

(b) Calculate the percentage error made in the approximation in (a) above.

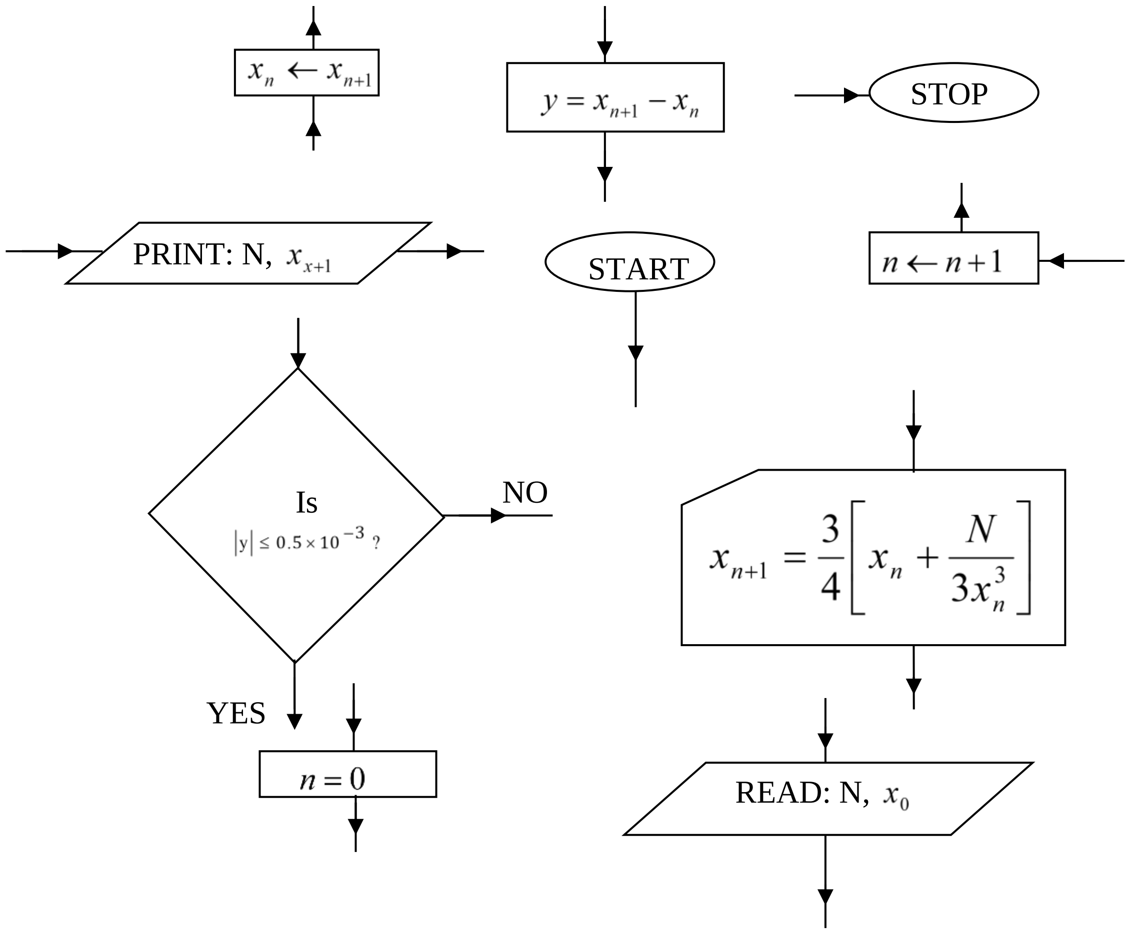
19. (a) Two iterative formulae A and B shown below are used to solve the equation f(x) =0. Only one of these re-arrangements provides a formula which converges to the root.

A	B
$x_{n+1} = \frac{1}{3}(x_n^3 + 1)$	$x_{n+1} = \frac{3x_n - 1}{x_n^2}$

(i) Determine the equation whose root is being sought.

(ii) Starting with $x_0 = 0.2$, use the most suitable formula to find the root of the equation f(x) =0 correct to 2 decimal places.

(b) Given below are elements of a flow chart not rearranged in order



(i) Re-arrange the elements to form a logical flow chat. State the purpose of the flow chart.

(ii) Perform a dry run of your flow chart, taking N= 187.42 and $x_0 = 3.0$. Give your answer correct to three decimal places.

Important information:

Format for the presentations:

(a) All schools will receive the same questions and will be required to select questions and book with the organisers their preferred questions for discussion through the WhatsApp platforms.

(b) Each presenter selected to present at the seminar will prepare their presentation and type it out if possible or write it down in a good handwriting with a black pen. These presenters should be supported by their subject teachers where possible. These final presentations will be sent to the organisers to be uploaded on the HeLP site before the seminar day- www.help.sc.ug .

(c) On the seminar day within the schools:

i. Each school will designate a presentation room that will accommodate the team making the presentation and answering questions from the audience. This room will have all the camera work with a fine and long blackboard or white board and a support subject teacher and ICT back stopper.

ii. The school should also arrange for 2-3 other rooms where the rest of the students will sit while observing the health SOPs. Each room should have a projector and laptop connected to the internet with a solid sound system. The school could use a big hall and arrange 3 screens all connected to laptops.

iii. All the teachers of mathematics in the school should be encouraged to attend the seminar to support the students in following keenly the presentations.

iv. On the seminar day, the expert presenter will make a presentation(15min) and answer the questions raised from the audience(10min)

v. The teachers will also make any additional comments(5min)

vi. A time will be provided between presentations to receive comments from UCC, RENU and other partners.

The Holistic eLearning Platform (HeLP) is inviting you to a scheduled Zoom meeting.
Topic: A-LEVEL MATHEMATICS VIRTUAL SEMINAR
Time: Feb 27, 2021 08:30 AM Nairobi

Join Zoom Meeting
https://us02web.zoom.us/j/81410189011?pwd=Rit6YzJCNzA3RzdMOFMrNnJyTUdyZz09

Meeting ID: 814 1018 9011
Passcode: 940152

APPLIED MATHEMATICS REVISION QUESTIONS

STATISTICS.

1. Consider the following data

CI	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 - 39
F	2	3	6	2	4	1

i) Draw the histogram of the data

ii) Determine the mode from your histogram or otherwise

iii) Calculate the mean distribution using the assumed formula

2. The table below shows the marks obtained by students of mathematics in a certain school.

Marks	Number of students
30 - < 40	2
40 - < 50	15
50 - < 55	10
55 - < 60	11
60 - < 70	30
70 - < 90	29
90 - < 100	3

i) Calculate the mean, median and standard deviation

ii) Draw a histogram for the data, hence determine the modal mark

iii) Draw an ogive, hence determine the median from the ogive and compare it with the calculated value.

3. Consider the following data below

137	140	150	140	157	131
141	142	162	169	166	129
138	170	170	152	161	139
122	131	139	170	165	145
140	147	125	134	153	145
151	128	129	121	154	167
122	165	128	149	140	136
130	170	133	139	136	150

Construct a frequency table with classes of equal width starting with 121 – 130 and use it to calculate the mean, mode and standard deviation

4. Consider the following data

CI	10 – 14	15 – 19	20 – 29	30 – 34	35 – 44	45 - 49
f	2	3	6	2	4	1

i) Draw the histogram of the data

ii) Determine the mode from your histogram

iii) Calculate the quartile deviation

5. a) In 1995 the prices of commodities A, B and C were shs. 720, shs. 830 and shs. 950 respectively. Given that the prices after 5 years were shs. 860, shs. 940 and shs. x and the simple aggregate price index number was 140. Find x

b) The price of a radio in 1995 was shs. 30,000. The index number for the price of a radio in 1985 was 1.6 based on 1975. 1n 1995, it was 0.75 based on 1985. Calculate

i) The prices of the radio in 1975 and 1985

ii) The price relative in 1995 based on 1975

6. A pharmacist had the following record of unit price and quantities of drugs sold for the years 1990 and 1991

Drug	Quantities in cartons		Unit price per carton	
	1990	1991	1990	1991
Aspirin	40	45	80	125
Panadol	70	90	100	90
Quinine	08	10	55	70
flue caps	10	10	90	100

Using 1990 as the base year, calculate

i) The price relative for each drug

ii) The simple aggregate price index number for 1991

iii) Fisher's index number for 1991

7. The table below shows the performance of 100 students in a resource examination

Score (%)	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49	50 - 59
Candidates	10	x	25	30	y	10

i) Given the median is 30.5, determine the values of x and y

ii) Hence calculate the average score and the mode

iii) Construct a cumulative frequency curve and use it to estimate the inter-quartile range

8. Using the figures in the table below

Food	2000		2005	
	Quantity in kg	Price per kg(shs)	Quantity in kg	Price per kg(shs)
Maize	20	650	25	700
Wheat	10	1500	8	1600
Beans	5	150	8	200

Calculate

i) Paasche aggregate price index

ii) Laspeyre aggregate price index

9. The cost of making a cake is calculated from the baking flour, sugar, milk and eggs. The table below gives the cost of these items in 1990 and 2000

Item	1990	2000	Weight, w
Flour per kg	600	780	12
Sugar per kg	500	400	5
Milk per litre	250	300	2
Eggs per egg	100	150	1

Using 1990 as the base year, calculate

i) The price relative for each item. Hence find simple price index for the cost of making a cake

ii) The simple aggregate price index number for 2000

iii) Fisher's index number for 2000

10. The following items are used in 1990 and 1985 as shown in the table below

Item	1985 price (£)	1990 price (£)	Weight, w
Radio	56	60	4
Shoes	45	50	2
Cap	15	20	1

Calculate the weighted price index number using 1985 as a base year

11. Given that A and B are mutually exclusive events such that P(A) = 0.3 and P(AUB) = 0.7. Find

i) P(B) ii) $P(A^1 \cap B)$

iii) $P(A^1 U B)$ iv) $P(A^1 \cap B^1)$

12. Given two events A and B such that $P(A^1) = {}^2/_3$, $P(B) = {}^1/_2$, and $P(A \cap B) = {}^1/_{12}$. Find

i) P(A) ii) P(AUB)

iii) $P(A^1 \cap B)$ iv) $P(A^1 U B^1)$

13. Events A and B are such that $P(A) = {}^1/_2$, $P(B) = {}^3/_8$, and $P({}^A/_B) = {}^7/_{12}$. Find

i) $P(A \cap B)$ ii) $P({}^B/_A)$

iii) $P(A^1 \cap B)$ iv) $P\left({}^B/_{A^1}\right)$

14. Two independent events A and B are such that P(A) = 0.4, P(B) = b and P(AUB) = 0.7. Find

i) The value of b ii) $P(A \cap B)$

iii) $P(A \cap B^1)$ iv) $P(A \, U \, B^1)$

15. There are 3 black and 2 white balls in each of two bags. A ball is taken from the first bag and put in the second bag, and then a ball is taken from the second bag into the first bag. What is the probability that there are now the same numbers of black and white balls in each bag as there were to begin with?

16. A fair coin is tossed four times. Use the tree diagram to calculate the probability of obtaining

    i) Exactly three heads

    ii) At least two tails

    iii) Exactly one head

    iv) Not more than three tails

17. The probabilities that a man makes a journey by car, air and road are respectively $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$. If the probabilities of an accident occurring when he uses these means of transport are $\frac{1}{3}, \frac{3}{10}$ and $\frac{1}{10}$ respectively

    i) What is the probability of an accident occurring

    ii) If the accident is known to have happened, find the probability that the man was travelling by Air

    ii) If it is known that the man reached safely, find the probability that he used the road

18. Three workers of dairy cooperation, Joy, Jane and Juliet seal milk sackets. On a particular day Joy seals 48%, Jane 30% and Juliet 22%, the probability that Joy seals wrongly is 0.53, Jane seals wrongly is 0.30 and Juliet seals wrongly is 0.17. Find the probability that a sacket was sealed wrongly and a wrongly sealed sacket found by the checker was sealed by Joy.

19. A bag contains white, yellow and blue beads in the ratio of 3: 4: 1. Two beads are selected at random without replacement, one after the other, obtain the probability that

    i) Two of the selected beads are of the same colour

    ii) The selected beads of different colours

20. A committee of 5 is selected from 4 men and 3 women.

    i) In how many ways can this be done if there must be more men than women?

    ii) What is the probability that the committee consists of 2 men

    iii) A woman is selected at random, find the probability that she belongs to this committee.

21. A discrete random variable X has a pdf, f(x) given by

$$f(x) = \begin{cases} k(x+1), & x = 1, 2, 3, 4. \\ kx, & x = 5, 6, 7 \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) The value of a constant, k

ii) $P(2 \leq x < 7)$

iii) The mean and mode of X

iv) The standard deviation of X

v) The semi inter-quartile range

22. A random variable X has the following cdf, F(x) given by

$$F(x) = \begin{cases} \frac{kx}{4}, & x = 3, 4, 5 \\ k(x-3), & x = 6, 7, 8 \\ 1, & x \geq 8 \end{cases}$$

Find

i) The value of the constant, k

ii) The pmf of x

iii) Expectation of X

iv) $P(x \geq E(x))$

v) Median and variance of X

23. The number of days the machine breaks down in a week follows a discrete random variable X with the following pdf, f(x) given by

$$f(x) = \begin{cases} kx^2, & x = 1, \ldots, 4. \\ k(7-x)^2, & x = 5, 6 \\ \frac{k}{7}, & x = 7 \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) The value of a constant, k

ii) The mean number of days the machine breaks down in a week

iii) The probability that the machine breaks down not more than 3 days in a week

24. A discrete random variable X has a probability density function, f(x) given by

$$f(x) = \begin{cases} \frac{x}{k}, & x = 1, 2, \ldots, n \\ 0, & \text{elsewhere} \end{cases}$$

Given that the expectation of X is 3. Find

i) The value of n and the constant, k

ii) The median of X

iii) The variance of X

iv) $P(x = 2/x \geq 2)$

v) The cdf of X

25. The number of days the machine breaks down in a month follows a discrete random variable X with the following pdf, f(x) given by

$$f(x) = \begin{cases} k\left(\frac{1}{4}\right)^x, & x = 0, 1, 2, \ldots \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) The value of a constant, k

ii) The probability that the machine breaks down not more than 2 times in a month

26. a) The probability distribution of a discrete random variable X is given by

$$P(X = r) = \begin{cases} kr, & r = 1, 2, 3, \ldots, n \\ 0, & \text{elsewhere} \end{cases}$$

i) Show that the constant, $k = \frac{2}{n(n+1)}$

ii) Find in terms of n the mean and variance of X

b) The discrete random variable X has a probability density function given by

$$f(x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1, 2, 3, 4, 5 \\ C, & x = 6 \\ 0, & \text{otherwise} \end{cases}$$

Determine

i) The value of the constant, C

ii) The mean of X

iii) The mode of X

27. A random variable X has the following probability distribution

$P(X = -2) = P(X = -1) = 2P(X= 0)$, $P(X = 1) = 0.2$, $2P(X = 2) = P(X =3)$

The mean of X equals the probability with which X assumes the value of -1. Find

i) $P(X = 2)$ and $P(X = 0)$

ii) $P(x \le 2/x \ge 0)$

iii) The standard deviation of X

iv) The upper quartile

28. The random variable X takes integer values only and has a pdf given by

$$P(X = x) = \begin{cases} kx, & x = 1, 2, 3, 4, 5 \\ k(10 - x), & x = 6, 7, 8, 9 \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) The value of the constant, k

ii) $E(X)$ and $Var(X)$

iii) $E(2X - 3)$ and $Var(2X - 3)$

iv) $P(2 \le 2X - 2 \le 11)$

29. Two random variables X and Y take on integer values with probabilities as given below

X	2	3	4	5	6
P(X = x)	0.25	0.15	0.10	0.30	0.20

Y	-1	0	1	2	3
P(Y = y)	0.20	0.25	0.40	0.10	0.05

Find;

i) $E(2X - 4)$

ii) $Var(2X + Y)$

iii) $Var(3X - 4Y)$

iv) $Var(X + Y)$

30. A discrete random variable X has the following distribution given by

X	1	2	3	4	5
P(X = x)	$\frac{4}{15}$	$\frac{3}{40}$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{11}{24}$

Two other random variables Y and Z are defined in terms of X as follows

Y = 2X + 1 and Z = 3X – 2

Find

i) The pmf of Y and Z ii) Standard deviation of Z

iii) The variance of Y iv) Draw the graph of cdf of Z

v) $P(3 < Z < 9)$

vi) Plot a graph of pmf of Y use it to find the mode

31. A random variable X has pdf, f(x) where

$$f(x) = \begin{cases} k[2 - (x+1)^2], & 1 \le x \le 3 \\ 0, & \text{elsehere} \end{cases}$$

Determine the

i) The value of a constant

ii) The c.d.f, F(x) of X

32. A random variable X has p.d.f f(x) given by

$$f(x) = \begin{cases} kx, & 0 \le x \le 2 \\ 2k(x-1)^2, & 2 \le x \le 5 \\ 0, & \text{elesewhere} \end{cases}$$

Determine the

i) Value of a constant, k ii) mean of X

iii) Standard deviation of X iv) $P(|x - 3| < 1)$

33. A random variable X has pdf f(x) given by

$$f(x) = \begin{cases} kx, & 0 \le x \le 1 \\ \frac{k}{2}(x+1), & 1 \le x \le 3 \\ 2k, & 3 \le x \le 4 \\ 0, & \text{elesewhere} \end{cases}$$

i) Sketch a graph of f(x) and use it to find ii) The constant, k

iii) Calculate the cdf, F(x) of X iv) $P(1 \le x \le 4)$

v) Calculate the mean and standard deviation of x

34. A random variable X has the following cdf, F(x) given by

$$F(x) = \begin{cases} 0, & x \le 0 \\ \frac{k}{2}x^2, & 0 \le x \le 1 \\ \frac{k}{2} + \frac{k}{4}(6x - x^2 - 5), & 1 \le x \le 3 \\ 1, & x \ge 3 \end{cases}$$

Find the

i) Value of the constant, k

ii) The pdf, f(x) of X

iii) $P(1 \le 2x \le 3)$

iv) Sketch the graph of f(x)

v) $P(0 \le x \le 2/0.5 \ge x)$

35. A random variable X has probability density function, f(x) given by

$$f(x) = \begin{cases} \frac{2}{3}a(x + a), & -a \le x \le 0 \\ \frac{1}{3a}(2a - x), & 0 \le x \le 2a \\ 0, & \text{elesewhere} \end{cases}$$

Determine the

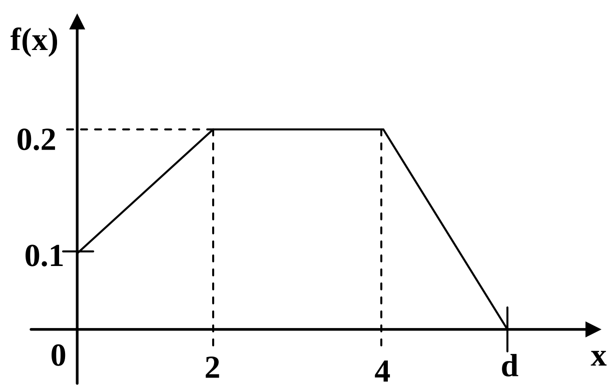
i) Value of the constant, a

ii) median of X

iii) $P(x \le 1.5/x > 0)$

iv) Cumulative distribution function, F(x) and sketch it

36. The motion of a motorist may be modeled into a continuous random variable X with the graph of f(x) as shown below;



Calculate the

i) value of d

ii) pdf, f(x) of X

iii) standard deviation

iv) cdf, F(x) of X

v) the median of X

37. A continuous random variable X has a pdf given by

$$f(x) = \begin{cases} kx, & 0 \le x \le 1 \\ k, & 1 \le x \le 2 \\ 2(3-x), & 2 \le x \le 3 \\ 0, & \text{elesewhere} \end{cases}$$

i) Sketch f(x) and use it to find the value of k

ii) Calculate the mean of X iii) Find $P(x < 2.5)$

iv) Determine the cumulative distribution function, F(x)

38. A continuous random variable X has a probability function, f(x) given by

$$f(x) = \begin{cases} \frac{1}{3}(x-2), & 2 \le x \le 3 \\ a, & 3 \le x \le 5 \\ 2-bx, & 5 \le x \le 6 \\ 0, & \text{elesewhere} \end{cases}$$

i) Find the value of a and b

ii) Determine $P(x > \frac{11}{2})$

iii) Determine the inter-quartile range

iv) Obtain the expression for F(x)

39. A continuous random variable X has a probability function, f(x) given by

$$f(x) = \begin{cases} kx(3-x), & 0 \le x \le 2 \\ k(4-x), & 2 \le x \le 4 \\ 0, & \text{elesewhere} \end{cases}$$

Find

i) The value of k iii) The mean of X

ii) $P(1 < x < 3)$

iii) Cumulative distribution function, F(x)

40. A continuous random variable X has a probability function, f(x) given by

$$f(x) = \begin{cases} kx, & 0 \le x \le 1 \\ k(4-x^2), & 1 \le x \le 2 \\ 0, & \text{elesewhere} \end{cases}$$

Find

i) The value of k

ii) E(X) and Var (x)

iii) Cumulative distribution function, F(x)

41. A continuous rectangular random variable X has limits 3 and 8

i) Draw the graph of its pdf

ii) Use the graph to find $P(x > 5)$

iii) Find the cdf and draw its graph

iv) Find the median of X

42. The continuous random variable X has the pdf, f(x) given by

$$f(x) = \begin{cases} \frac{1}{k}, & 24 \le x \le 34 \\ 0, & Otherwise \end{cases}$$

Find the

i) Value of k

ii) Mean, median and quartile deviation

iii) Cdf of X

iv) Value of a if $P(x > a) = 0.65$

43. It is known that a continuous random variable X has uniform distribution over the interval [20, 30]. Find the

i) Mean and standard deviation of X

ii) Median and quartile deviation of X

iii) $P(25 < x < 29/x > 27)$

iv) cdf of X

44. A continuous random variable X has uniform distribution over the interval [d, e]. d < e. Show that

i) The mean is equal to the median

ii) $SD(x) = \frac{e - d}{2\sqrt{3}}$

iii) If $E(x) = 3$ and $Var(x) = \frac{4}{3}$ then $e = 5$ and $d = 1$

45. A discrete random variable X has uniform distribution over the interval [a, b].

a < b, a and b are integers

i) Show that its pdf satisfies properties of the pdf of a discrete random variable

ii) Prove that its mean is $\frac{1}{2}(a + b + 1)$

iii) Obtain the expression for the variance hence its SD

iv) Find its median and quartile deviation if $a = 10$ and $b = 25$

MECHANICS

1. A train approaching a station covers successive half kilometers in 16s and 20s respectively. Assuming the retardation to be uniform, find the further distance the train run before coming to a stop.

2. Two points P and Q are x m apart. A boy starts from rest at P and moves directly towards Q with an acceleration of a ms^{-2} until he acquires a speed of Vms^{-1}. He maintains this speed for T seconds and then brought to rest at Q under a retardation of a ms^{-2}. Prove that $T = \frac{x}{V} - \frac{V}{a}$

3. A car started from rest accelerated uniformly for 2 minutes and then maintained a speed of $50kmh^{-1}$. Another car started 2 minutes later from the same spot and this car too accelerated uniformly for 2 minutes and it then maintained a speed of $75kmh^{-1}$.

   i) Draw a velocity – time graph and find when and where the second car overtook the first.

   ii) The car maintained the speed of $50kmh^{-1}$ for 10minutes. It then decelerated uniformly for further 2½ minutes before coming to rest. How far has the car travelled from the start?

4. A motor car A passes a certain point P with a speed of $7.5ms^{-1}$ and an acceleration of $0.3ms^{-2}$. Five seconds later a car B passes P with a speed of $15.0ms^{-1}$ and an acceleration of $0.2ms^{-2}$. Prove that if their maximum speed is $30.0ms^{-1}$, B will ultimately be travelling 131m a head of A.

5. A car starting from rest is uniformly accelerated during the first 0.5km of its run, then run 1.5km at a uniform speed and is afterwards brought to rest in ¼km under uniform retardation. If the time for the whole journey is 5minutes, find the uniform acceleration and uniform retardation.

6. A particle starts from rest and moves in a straight line with uniform acceleration. In 4 seconds of its motion it travels 12m and in the next 5 seconds it travels 30m. Find its

   i) Acceleration

   ii) Final velocity

7. A train travels a long a straight track between two stations A and B. the train starts from rest at A and accelerates at $1.25ms^{-2}$ until it reaches a speed $20ms^{-1}$. It then travels at this speed for a distance of 1.56km and then decelerates at $2ms^{-2}$ to come rest at B.

i) Sketch a velocity-time graph for the motion of the train

ii) Find the distance from A to B

iii) Find the total time of the journey

8. A, B and C are three points which lie in that order on a straight road with AB = 95m and BC = 80m. a car is travelling along the road in the direction ABC with constant acceleration of ams^{-2}. The car passes through A with speed u ms^{-1} reaches B later and C two seconds after that. Find the values of a and u.

9. Two stations A and B are a distance of 6x mtres apart along a straight line. A train starts from rest at A and accelerates uniformly to a speed of Vms^{-1} covering a distance of x metres. The train then maintains this speed until it has travelled a further 3x metres. It then retards uniformly to rest at B. sketch a velocity- time graph for the motion of the car and show that if T is the time taken to travel from A to B then T = $\frac{9x}{V}$ seconds.

10. P, Q and R are points on a straight road such that PQ = 20m and QR = 55m. a cyclist moving with uniform acceleration passes P and then notices that it takes him 10s and 15s to travel between (P and Q) and (Q and R) respectively. Find his uniform acceleration.

11. A body is projected vertically upwards with a velocity of $25ms^{-1}$. Find

i) How high it will go

ii) What times elapses before it is at a height of 20m

iii) The time to reach maximum height hence time of flight

12. A ball is thrown vertically upwards with a speed of $42ms^{-1}$. If it falls past the point of projection into a sea of depth 80m, find when it strikes the bottom of the sea.

13. A particle is projected from a point O with an initial velocity 3i + 4j. Find in vector form the velocity and position of the particle at any time, t.

14. A stone is thrown vertically upwards with a velocity of $25ms^{-1}$, if another stone is thrown vertically upwards 4 seconds later with the same speed from the same point of projection. Determine when and where the two stones meet.

15. A stone is dropped from the top of the building and at the same instant another stone is thrown vertically upwards from the bottom of the building with a speed of $20ms^{-1}$. They pass each other three seconds later. Find the height of the building.

16. A particle is projected vertically upwards with a certain velocity and it is found that when it is 400m from the ground it takes 8 seconds to return to the same point again. Find the velocity of projection and the time of flight.

17. The ball is thrown vertically down wards from the top of the tower and has an initial speed of $4ms^{-1}$. If the ball hits the ground 2 seconds later. Find

    i) the height of the tower

    ii) the speed at which the ball strikes the ground.

18. A stone is thrown vertically upwards from the ground level at a speed of $24.5ms^{-1}$. Find how long after projection the stone is at a height of 19.6m above the ground for the first time and the second time and how long is the stone at least 19.6m above the ground level.

19. A ball is thrown vertically upwards with a speed, u after time, t another ball is thrown vertically upwards from the same point with the same speed. Prove that they will meet after elapse of $\left(\frac{t}{2}+\frac{u}{g}\right)$ seconds from the time the first particle was projected hence show that the distance travelled is $\left(\frac{4u^2-g^2t^2}{8g}\right)$ m.

20. A particle is projected vertically upwards from a point O with a speed $\frac{4}{3}$ V. after it has travelled a distance $\frac{2}{5}$ x above O on its upward motion, a second particle is projected vertically upwards from the same point and with the same initial speed. Given that the particles collide at a height $\frac{2}{5}$ x above O, x and V being constant, show that

i) at maximum height H, $8V^2 = 9gH$

ii) when the particles collide $9x = 20H$

21. Find the angle between a and b, given that

$a = 5i + 6j + k$ and $b = 2i + j$

22. A particle of mass 5kg at rest at appoint (1, -4, 4) is acted upon by three forces $F_1 = 3i + 3j$, $F_2 = 2j + 4k$ and $F_3 = 2i + 6k$. Find

i) Acceleration of the particle

ii) Velocity and speed of the particle after 4 seconds

iii) Position and the distance of the particle after 4 seconds

23. The forces $F_1 = 2i + 3j$, $F_2 = i + 3j$ and $F_3 = i + 2j$ act on a particle of mass 2kg located at (1, -1). Find

i) The magnitude and direction of the resultant force

ii) Position and the distance of the particle after 4 seconds

24. Three forces $F_1 = 6i + 3j$, $F_2 = -2i + 3j$ and $F_3 = \lambda i$ act on a particle at the origin, if the magnitude of the resultant is 10N. Calculate the two possible values of λ and the two possible directions of the line of action of the resultant.

25. A particle of mass 3kg moving on the curve described by

$r = 4\sin 3t i + 8\cos 3t j$ where r is the position vector at time, t.

i) Determine the position and the velocity of the particle at time, t = 0s

ii) Show that the force acting on the particle is $-27r$.

26. A particle of mass 2kg initially at rest at (0, 0, 0) is acted upon by the force $\begin{pmatrix} 2t \\ t \\ 3t \end{pmatrix}$ N. Find

i) Acceleration and velocity after 3 seconds.

ii) Distance moved after 3 seconds.

27. A particle of weight 8N is attached at a point B of a light inextensible string AB. It hangs in equilibrium with point A fixed and AB at an angle of 30^o to the vertical. A force, F at B acting at right angles to AB keeps the particle in equilibrium. Find the magnitude of F and the tension in the string.

28. A particle of mass 3kg is attached to the lower end B of an inextensible string. The upper end A of the string is fixed to a point on the ceiling of a roof. A horizontal force of 22N and an upward vertical force of 4.9N act upon the particle making it be in equilibrium with the string making an angle, $\propto$ with the vertical. Find the value of $\propto$ and the tension in the string.

29. A non-uniform rod of mass 9kg rests horizontally in equilibrium supported by two light inextensible strings tied to the ends of the rod. The strings make angles of 50^o and 60^o with the rod. Calculate the tensions in the strings.

30. A particle moving with an acceleration given by

$a = 4e^{-3t}i + 12\sin tj - 7\cos tk$ is located at a point (5, -6, 2) and has velocity $v = 11i - 8j + 3k$ at time $t = 0$. Find the

i) Magnitude of the acceleration when $t = 0$

ii) Velocity at any time, t

iii) Displacement at any time, t

31. A particle with position vector $10i + 3j + 5j$ moves with constant speed of $6ms^{-1}$ in the direction $i + 2j + 2k$. Find its distance from the origin after 5 seconds.

32. A particle of mass 2 units moves under the action of force which depends on time, t given by $F = 24t^2 i + (36t - 16)j$. given that t = 0, the particle is located at $3i - j$ and has a velocity $6i + 15j$. Find

i) the kinetic energy of the particle at t = 2

ii) the impulse in moving the particle from t = 1 to t = 2 s

33. If the $a = 6\sin 6ti + 9\cos 3t\, j$. find displacement when $t = \frac{1}{3}$▯ given that $v = i + 3j$ and $s = 5i + 2j$ when $t = \frac{\pi}{6}$

34. An object of mass 5kg is initially at rest at a point position vector is $-2i + j$. if it is acted upon by a force $F = 2i + 3j - 4k$. Find

i) the acceleration

ii) the velocity after 3 seconds

iii) the distance from the origin after 3 seconds.

35. A particle of mass 2m rests on a rough plane inclined at an angle of $\tan^{-1}(3\mu)$ where μ is the coefficient of friction between the particle and the plane. The plane is acted upon by a force of PN

a) Given that the force acts along the line of greatest slope and that the particle is on a point of sliding up. Show that the maximum force possible to maintain the particle in equilibrium is

$$P_{max} = \frac{8\mu mg}{\sqrt{1 + 9\mu^2}}$$

b) Given that the force acts horizontally in a vertical plane through the line of greatest slope and that the particle is on a point of sliding down the plane. Show that the force required to maintain the particle in equilibrium is

$$P = \frac{4\mu mg}{1 + 3\mu^2}$$

36. A carton of mass 3kg rests on a rough plane inclined at an angle of 30^0 to the horizontal. The coefficient of friction between the carton and the plane is ⅓. Find a horizontal force that should be applied to make the carton just about to move up the plane.

37. A body of mass 8kg rests on a rough plane inclined at θ to the horizontal. If the coefficient of friction is μ, find the least horizontal force in terms of μ, θ and g which will hold the body in equilibrium.

38. A particle of mass 2kg rests on a rough inclined plane at an angle $\sin^{-1}\left(\frac{5}{13}\right)$. A force of 8N acts on the particle along the line of greatest slope

    i) given that the particle is about to move up the plane, calculate the coefficient of friction between the particle and the plane

    ii) if the 8N force is removed, find the acceleration of the particle down the plane.

39. A box of mass 2kg is at rest on a plane inclined at 25^o to the horizontal. The coefficient of friction between the box and the plane is 0.4. What minimum force applied parallel to the plane would move the box up the plane.

40. A particle of mass ½ kg is released from rest and slides down a rough plane inclined at 30^o to the horizontal. It takes 6 seconds to go 3m.

    i) find the coefficient of friction between the particle and the plane

    ii) what minimum horizontal force is needed to prevent the particle from moving.

41. A particle of weight, W is at rest on an inclined plane under the action of a force, P acting parallel to the line of greatest slope of the plane in an upward direction. The angle of friction between the particle and the plane is λ and the angle of inclination of the plane to the horizontal is 2λ. Show that $P_{max} = W\tan\lambda(4\cos\lambda - 1)$ and $P_{min} = \mu W$ respectively. Calculate the coefficient of friction between the particle and the plane.

42. A particle of mass 2kg rests on a rough horizontal ground. The coefficient of friction between the particle and the ground is ½. Find the magnitude of the force, P acting upwards on the particle at an angle of $30^o$to the horizontal which will just move the particle.

43. A parcel of mass 2kg is place on a rough plane which is inclined at an angle of 45° to the horizontal. The coefficient of friction between the parcel and the parcel and the plane is 0.25. Find the force that must be applied to the parcel in a direction parallel to the plane so that

i) the parcel is just prevented from sliding down the plane

ii) the parcel is just on the point of moving up the plane

iii) the parcel moves up the plane with an acceleration of $1.5ms^{-2}$

44. when at an angle of elevation, $\propto$ a gun fires a shot to hit a mark P on the horizontal plane, when the angle is reduced to 15° the shot falls 100m short of P but when the elevation is 45° it falls 400m beyond P. find the value of $\propto$ and distance of P from the gun.

45. The horizontal and vertical components of the initial velocity of a particle projected from a point O on the horizontal plane are p and q respectively.

i) Express the vertical distance Y travelled in terms of the horizontal distance X and the components of p and q.

ii) Find the greatest height, H attained and the range, R on the horizontal plane through O. hence show that $Y = \frac{4HX}{R^2}(R - X)$. Given that the particle passes through the point (20, 80) and H = 100m. Find the velocity of projection.

46. A particle P is projected from a point A with an initial velocity of $60ms^{-1}$ at an angle of 30° to the horizontal. At the same time and the same instant a particle Q is projected in opposite direction with initial speed $50ms^{-1}$ from a point at the same level with A and 100m from A. given that the particles collide, find

i) the angle of projection of q

ii) the time when collision occurs

47. If T is the time of flight and x the horizontal range of a projectile, prove that $gT^2 = 2x \tan \propto$. Where $\propto$ is the angle of projection

48. A projectile having a horizontal range, R reaches a maximum height, H. prove that it must have been launched

i) with an initial speed equal to $\left[\frac{g(R^2+16H^2)}{8H}\right]^{½}$

ii) at an angle with the horizontal given by $\sin^{-1}\left[\frac{4H}{(R^2+16H^2)^{½}}\right]$

49. A ball is kicked from a point O so that it just clears two trees which are of height, h and at a distances x and y respectively from O. prove that if θ is the angle of projection of the ball,

i) $\tan\theta = \frac{h(x+y)}{xy}$

ii) the maximum height of the ball, $H = \frac{h(x+y)^2}{4xy}$

50. A particle is projected so as it just clears to walls each of height, h which lie at right angles with the plane of motion. The walls are at a distance, d apart and the first wall is at a distance, L from the point of projection. Show that the angle of elevation of the particle, ∝ is given by

$\tan\propto = \frac{h(2L+d)}{L(L+d)}$

51. A particle is projected from the top of the vertical cliff 160m high with a velocity of $180ms^{-1}$ at an angle of elevation of 30^o. Find the horizontal distance from the foot of the cliff to the point where it strikes the ground.

52. A particle is projected from a point height 3h above a horizontal plane, the direction of projection making an angle, ∝ with the horizon. Show that if the greatest height above the point of projection is h, the horizontal distance travelled before striking the plane is 6hcot ∝.

53. Two particles are projected simultaneously from the top and the bottom of a vertical cliff with angles of elevation ∝ and β respectively. They strike an object at the same point simultaneously. Show that if p is the horizontal distance of the object from the cliff, the height of the cliff is given by p(tanβ - tan∝)

54. Two boys stand on a horizontal ground at a distance, d apart. One throws a ball from a height, 2h with a velocity, V and the other catches it at a height, h above the horizontal at which the first boy throws the ball. Show that $gd^2 \tan^2\theta - 2V^2 d\tan\theta + gd^2 - 2V^2 h = 0$ holds if $d = 2h\sqrt{2}$ and $V^2 = 2gh$. Hence calculate

i) the value θ

ii) the greatest attained by the ball in terms of h, u, g and θ

55. A bullet is fired out with the initial velocity of projection is $240ms^{-1}$ to the sea in a horizontal direction from a gun situated on the top of a cliff 78.4m high. Calculate

i) the distance at which the bullet will strike the water from the foot of the cliff.

ii) the inclination to the horizontal at which the bullet will strike the surface of the water.

56. a) A particle is projected at an angle of elevation of 30^o with a speed of $21ms^{-1}$, if the point of projection is 5m above the horizontal ground. Find the horizontal distance that the particle travels before striking the ground.

b) A boy throws a ball at an initial speed of $40ms^{-1}$ at an angle of elevation, ∝. Show taking $g = 10ms^{-2}$, that the times of flight corresponding to a horizontal range of 80m are positive roots of the equation $T^4 - 64T^2 + 256 = 0$.

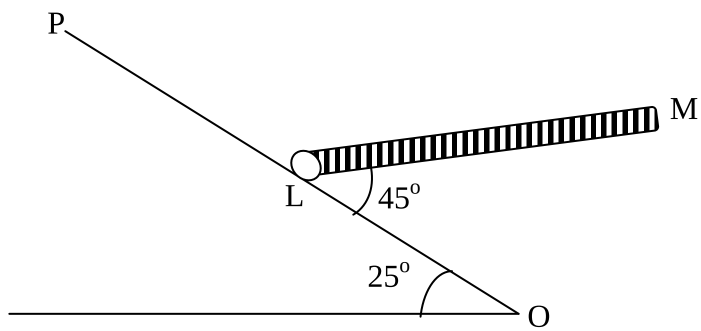
57. A particle is projected with a velocity of $40ms^{-1}$ at an angle of 60^o to the horizontal from the foot of the plane inclined at an angle of 30^o to the horizontal. Find the time at which the particle hits the plane.

58. A uniform ladder of length, 2L and weight, W rests in a vertical plane with one end against a rough vertical wall and the other end against a rough horizontal surface, the angles of friction at each end being $\tan^{-1}\frac{1}{3}$ and $\tan^{-1}\frac{1}{2}$ respectively.

i) if the ladder is in limiting equilibrium at either end. Find θ the angle inclination of the ladder to the horizontal.

ii) a man of weight 10 times that of the ladder begins to ascend it, how far will he climb before the ladder slips.

59. A uniform rod LM of weight, W rests with L on the smooth plane PO of inclination 25° as shown in the diagram below.



The angle between LM and the plane is 45°. What force parallel to PO applied at M will keep the rod in equilibrium? Give your answer in terms of W.

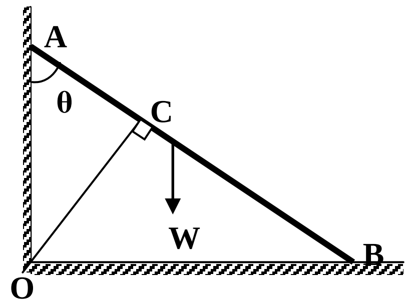
60. A non – uniform ladder AB, 10m long and mass 8kg lies in limiting equilibrium with its lower end A resting on a rough horizontal ground and the upper end B resting against a smooth vertical wall. If the Centre of gravity of the ladder is 3m from the foot of the ladder and the ladder makes an angle of 30° with the horizontal. Find the

i) coefficient of friction between the ladder and the ground

ii) reaction at the wall

61. Two smooth rods AB and AC each of weight, W and length 10cm are smoothly hinged at A. the ends B and C rest on a smooth horizontal plane. An extensible string joins B and C and the system is kept in equilibrium in a vertical plane with the string taut. An object of weight, 2W climbs the rod AC to a point E such that AE = 8cm. given that angle BAC is 2θ. Determine in terms of W and θ the reaction at the ends B and C and the tension in the string. Hence show that the reaction at the hinge A is given by $\frac{W}{10}\sqrt{49\tan^{-1}\theta + 4}$

62. The diagram below shows a uniform rod AB of weight, W and length, L resting at an angle, θ against a smooth vertical wall at A. The other end B rests on a smooth horizontal table. The rod is prevented from slipping by an inelastic string OC, C being a point on AB such that OC is perpendicular to AB and O on the point of intersection of the wall and the table. Angle AOB is 90°.

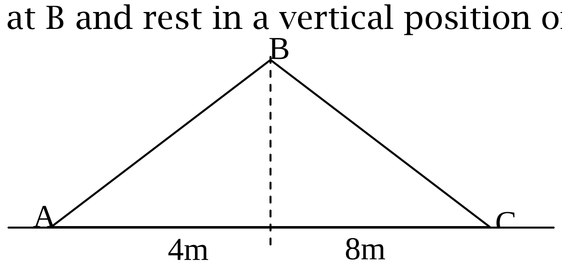


Find the

i) tension in the string

ii) reactions at A and B in terms of θ and W.

63. Two uniform rods AB and BC of masses 4kg and 6kg respectively are hinged at B and rest in a vertical position on the smooth floor as shown below.



A and C are connected by a rope

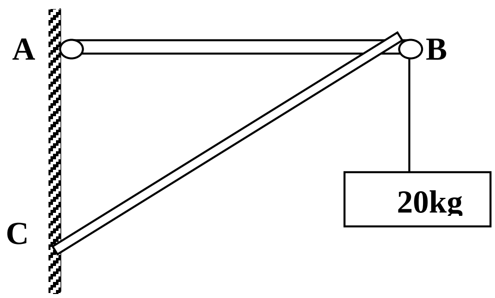
i) find the reactions between the rods and the floor at A and C when the rope is taut.

ii) if now a body is attached a quarter of the way up CB and the reactions are equal, find the mass of the body.

64. A non-uniform ladder AB is in equilibrium with A in contact with a horizontal floor and B in contact with a vertical wall. The ladder is in a vertical plane perpendicular to the wall. The Centre of gravity of the ladder is at G where AG = $\frac{2}{3}$AB. The coefficient of friction between the ladder and the wall is twice that between the ladder and the floor. If the

ladder makes an angle, θ with the wall and the angle of friction between the ladder and the floor is λ, prove that $4\tan\theta = 3\tan 2\lambda$. How far can a man of mass, m ascend the ladder without the ladder slipping given that $\theta = 45^o$ and the coefficient of friction between the ladder and the floor is ½.

65. The diagram below shows a uniform horizontal plank AB of length 3m and mass 2kg hinged to a vertical wall at A and supported at B by a light rod CB hinged to the same wall at C such that AC = 4m



If a mass of 20kg hangs from B, find the

i) tension force in the rod CB

ii) the magnitude of the reaction at A.

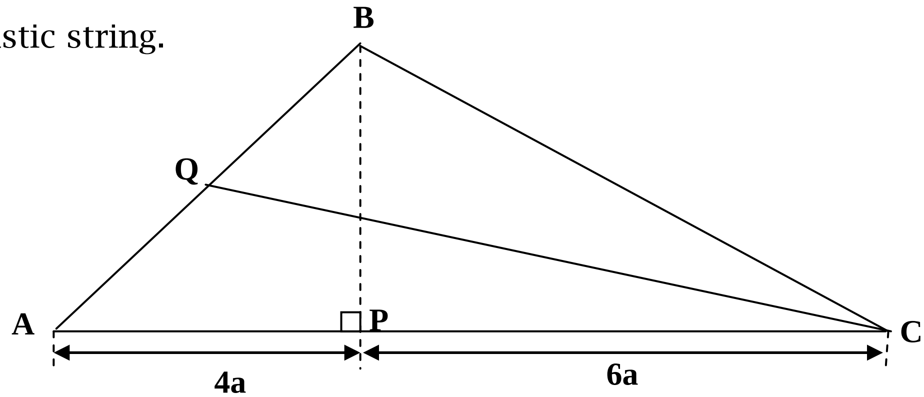
66. Two uniform rods AB and BC of equal length but of mass, M and 3M respectively are freely jointed together at B. the rods stand in a vertical plane with the ends A and C on a horizontal ground. The coefficient of friction, μ at the points of contact with the ground is the same and the rods are inclined at 60^o to each other. Given that one of the rods is on the point of sliding, find μ and the reaction at the hinge B when the rods are in this position.

67. A rod AB of length 0.6m long and mass 10kg is hinged at A. its centre of mass is 0.5m from A. a light inextensible string attached at B passes over a fixed smooth pulley 0.8m above A and supports a mass, M hanging freely. If a mass of 5kg is attached at B so as to keep the rod in a horizontal position, find the

i) value of M.

ii) reaction at the hinge.

68. The diagram below shows two uniform rods AB and BC of weights W and 3W respectively, which are smoothly hinged together at B. Point P is a point on AC which is vertically below B and AP = 4a, PC = 6a. The rods rest in equilibrium in vertical plane with the ends A and C on a smooth horizontal plane. The end C is connected to a point Q on AB by a light inelastic string.



Show that the magnitude of the reaction of the plane at A is $\frac{17}{16}$W and find the magnitude of the reaction of the plane at C. if BC = 10a and Q is the midpoint of AB, find the tension in CQ.

69. A non-uniform ladder AB of length 12m and mass 30kg has its Centre of gravity at the point of trisection of its length, nearer to A. the ladder rests with end A on the rough horizontal ground (coefficient of friction ¼) and B against a rough vertical wall (coefficient of friction $^1/_5$. The ladder makes an angle, θ with the horizontal such that tan θ = $\frac{9}{4}$. A straight horizontal string connects A to a point at the base of the wall vertically below B. what tension must the string be capable of withstanding if a man of mass 90kg is to reach the top of the ladder safely.

70. A non-uniform metallic beam AB of mass 30kg and length 4.4m balances with 45kg mass placed at end B. when support Q is placed 1.2m from B, find how far from end A the weight of the beam acts. If the beam balances again when an additional mass of 22.3kg is hang at end A and another support P is placed 0.8m from end A, determine the reactions at P and Q.

NAMILYANGO COLLEGE

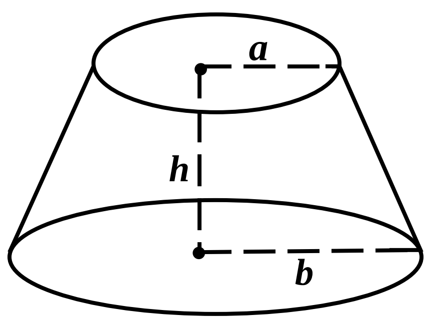
A–LEVEL APPLIED MATHEMATICS SEMINAR 2024

PAPER STRUCTURE	***SECTION A***	***SECION B***
Statistics and probability	***3***	***3***
Numerical methods	***2***	***2***
Static Mechanics	***1***	***1***
Dynamic Mechanics	***1***	***1***
Kinematic Mechanics	***1***	***1***

STATIC MECHANICS

1(a) *Show that the centre of gravity of a solid cone of radius* **r** *and height* **h** *lies along its axis at a distance* $\frac{1}{4}h$ *from the base*

(b) *The figure below shows a solid conical frustum of height* **h** *and whose top and bottom radii being* **a** *and* **b** *respectively*



Show that the centre of gravity of this frustum lies along its axis at a distance

$$\frac{h(b^2+2ab+3a^2)}{4(b^2+ab+a^2)}$$ *from the bottom*

2. (a) *A particle of weight* **50N** *is supported by two light inextensible strings of lengths* **8m** *and* **13m** *attached to two fixed points* **15m** *apart on a horizontal beam. Find the tension in each string*

(b) *The ends* ***P*** *and* ***Q*** *of a light inextensible string* ***PBCQ*** *are fastened to two fixed points on a horizontal beam. Particles of mass* ***3kg*** *and* ***4kg*** *are attached to the string at the points* ***B*** *and* ***C*** *respectively. If* ***PB*** *is inclined at* $45°$ *to the horizontal and* $\angle PBC = 150°$, *find the:*

(i) *tension in each portion of the string*

(ii) *angle* ***CQ*** *makes with the horizontal*

3. ABCDE *is a regular pentagon of side* ***4m.*** *Forces of magnitude* ***2N, 3N, 5N*** *and* ***7N*** *act along* $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ *and* $\overrightarrow{EB}$ *respectively. The resultant of this system of forces cuts* ***AB*** *produced at* ***H.*** *Taking* ***A*** *as the origin and* ***AB*** *as the* ***x–axis,***

(i) *find the magnitude and direction of the resultant force*

(ii) *show that length* $AH = 15\cdot 34m$ *correct to* ***4 S·f***

(iii) *find the perpendicular distance from* ***A*** *to the line of action of the resultant force*

4. *Coplanar forces* $(3i + 3j)N$, $(4i - 5j)N$, $(-5i + 2j)N$ *and* $(2i + 3j)N$ *act at points with position vectors* $(3i + j)m$, $(i + 3j)m$, $(-2i + j)m$ *and* $(-2i - 2j)m$ *respectively.*

(i) *Find the resultant force and find where its line of action cuts the* ***x–axis***

(ii) *A couple of moment* ***bNm*** *acting anticlockwise and a force* $(pi + qj)N$ *acting at a point with position vector* $(2i + j)m$ *are now added to the above system. If these reduce the system to equilibrium, find the values of* ***p, q*** *and* ***b***

5. *A uniform ladder* **PQ** *of length* **2a** *and weight* **w** *is inclined at an angle of* $\tan^{-1} 2$ *to the horizontal with its end* **Q** *resting against a smooth vertical wall and end* **P** *on a rough horizontal ground with which the coefficient of friction is* $\frac{5}{12}$. *If a boy of weight* **W** *can safely ascend a distance* **x** *up this ladder before it slips,*

(i) *show that* $x = \frac{a(2w + 5W)}{3W}$

(ii) *deduce that the boy can only reach the top of the ladder if* **W = 2w**

6. *A uniform rod* **PQ** *of length* **8m** *and weight* **18N** *is freely hinged at* **P** *and carries a mass of* **3kg** *at* **Q.** *The rod is kept horizontally by a string attached at* **Q** *and to a point* **C** *distant* **6m** *vertically above* **P.** *Find the:*

(i) *tension in the string*

(ii) *magnitude and direction of the reaction at the hinge*

7. *A box of mass* **6·5kg** *is placed on a rough plane inclined at* $\tan^{-1}\left(\frac{3}{4}\right)$ *to the horizontal. The coefficient of friction between the box and the plane is* **0·25.** *Find the least horizontal force required:*

(i) *to move the box up the plane*

(ii) *to prevent the box from sliding down the plane*

KINEMATIC MECHANICS

1. *Two cyclists **P** and **Q** are travelling along straight roads which cross at an angle of* $\boldsymbol{60°}$ *at point **C.** If their riding speeds towards **C** are* $\boldsymbol{4kmh^{-1}}$ *and* $\boldsymbol{5kmh^{-1}}$ *and they are respectively **8km** and **15km** from **C,** find the**:***

(i) *least distance between the cyclists*

(ii) *time that elapses before the cyclists are closest*

(iii) *distances of **P** and **Q** from **C** when they are nearest*

2. *A battleship and a patrol ship are initially **16km** apart with the battleship on the bearing of* $\boldsymbol{035°}$ *from the patrol**.** The battleship sails at* $\boldsymbol{14kmh^{-1}}$ *in the direction* $\boldsymbol{S30°E}$ *and the patrol ship at* $\boldsymbol{17kmh^{-1}}$ *in the direction* $\boldsymbol{N50°E}$***.***

(a) *Find the**:***

(i) *shortest distance between the ships*

(ii) *time that elapses before the ships are closest*

(b) *If the guns on the battleship have a range of up to **6km,** find the time that elapses when the patrol ship is within range of these guns*

3. *Two boats **P** and **Q** are sailing with respective speeds of* $\boldsymbol{20kmh^{-1}}$ *and* $\boldsymbol{19kmh^{-1}}$***.*** *Initially **P** is **10km** from **Q** on a bearing of* $\boldsymbol{320°}$ *and is on a course of* $\boldsymbol{200°}$***.***

*Find the**:***

(i) *two possible courses **Q** can take in order to intercept **P***

(ii) *time taken for interception to occur in each case*

4. (a) *At certain times, the position vector* **r** *and velocity vector* **v** *of two ships* ***A*** *and* ***B*** *are as follows:*

$$\boldsymbol{r_A = (-2i + 3j)km} \quad \boldsymbol{V_A = (12i - 4j)kmh^{-1}} \text{ at } \boldsymbol{11:45am}$$
$$\boldsymbol{r_B = (8i + 7j)km} \quad \boldsymbol{V_B = (2i - 14j)kmh^{-1}} \text{ at } \boldsymbol{12:00noon}$$

If the ships maintain these velocities, find the:

(i) *position vector of ship* ***A*** *at noon*

(ii) *time when the ships are closest*

(iii) *shortest distance between the ships*

(iv) *distance of ship* ***A*** *from the origin when the two ships are closest*

(b) *If instead ship* ***B*** *had a velocity* $\boldsymbol{V_B = (-2i - 14j)kmh^{-1}}$, *show that the ships will collide and find when and where the collision occurs*

5. *Two stations* ***P*** *and* ***Q*** *are* ***2·5km*** *apart. A train passes* ***P*** *at a speed of* $\boldsymbol{14ms^{-1}}$ *and accelerates uniformly for* ***20s*** *to a speed* $\boldsymbol{v_1}$. *Over the next* ***720m*** *covered in* ***15s,*** *its acceleration alters to a speed* $\boldsymbol{v_2}$. *It travels at this speed for* ***13s*** *and then over the next* ***500m*** *covered in* ***10s*** *with uniform deceleration its speed at* ***Q*** *is* $\boldsymbol{v_3}$. *Find the:*

(i) *values of* $\boldsymbol{v_1}$, $\boldsymbol{v_2}$ *and* $\boldsymbol{v_3}$

(ii) *acceleration for the second part of the motion*

(iii) *fraction of the whole distance covered with constant speed*

6. (a) *The velocity of a uniformly accelerating train changes from* $\boldsymbol{u}$ *to* $\boldsymbol{v}$ *in time* $\boldsymbol{t}$.

(i) *Sketch its velocity time graph*

(ii) *Derive the equation for its motion* $v^2 = u^2 + 2as$, *where* a *is its acceleration*

(b) *A uniformly accelerating train passes successive kilometer marks with velocities* $20ms^{-1}$ *and* $28ms^{-1}$ *respectively. Find its velocity when passing the next kilometer mark*

(c) *A uniformly retarding car takes* **8s** *and* **16s** *to travel between successive points* **A, B** *and* **C** *each* **144m** *apart. Find the further distance it travels to come to rest*

7. (a) *A particle is projected from level ground at an angle of elevation* θ *with initial speed* ums^{-1}. *Show that the equation of its path is given by*

$$y = x\tan\theta - \frac{gx^2(1+\tan^2\theta)}{2u^2}$$

(b) *A ball kicked from level ground with a speed of* $30ms^{-1}$ *just clears a vertical wall* **9m** *high and* **72m** *away. Calculate the possible angles of projection* (*use* $g = 10ms^{-2}$)

(c) *A ball projected at an angle with a speed of* $14\sqrt{10}ms^{-1}$ *from the top of a tower* **200m** *high hits the ground at a point* **200m** *away from the foot of the tower.*

(i) *Show that the two possible directions of projection are at right angles to each other*

(ii) *Find the two possible times of flight*

8. *A particle is projected from the origin* **O** *with velocity* $\boldsymbol{u} = (9\cdot 8\boldsymbol{i} + 29\cdot 4\boldsymbol{j})ms^{-1}$ *and moves freely under gravity.*

(a) *Find the particle's velocity and position vector after* **t** *seconds*

(b) *Show that the particle's equation of path is given by* $y = 3x - \dfrac{5x^2}{98}$.

Hence find the particle's horizontal range and maximum height reached

(c) *Find the direction in which the particle is moving after* **t** *seconds*

(d) *Find the two times when the direction in which the particle is moving is at right angles with the line joining the position of the particle to* **O**

DYNAMIC MECHANICS

1. *A force* $(24t\boldsymbol{i} - 12\boldsymbol{j})N$ *acts on a particle of mass* **2kg** *initially at rest at a point with position vector* $(-4\boldsymbol{i} + 3\boldsymbol{j})m$. *Find the:*

(i) *velocity of the particle after* **t** *seconds*

(ii) *distance from the origin after* **2s**

(iii) *power exerted by the force at* **t = 2s**

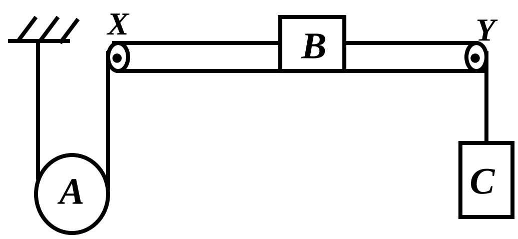
(iv) *work done by the force between* **t = 1s** *and* **t = 2s**

2. *A pile–driver of mass* m_1kg *falls through a height* **hm** *onto a pile of mass* m_2kg *without rebounding. If the pile is driven into the ground a depth* **d m,** *show that the resistance of the ground to penetration* $R = \dfrac{m_1^2gh}{(m_1 + m_2)d} + (m_1 + m_2)g$ *and the time for which the pile is in motion* $T = \dfrac{(m_1 + m_2)d\sqrt{2gh}}{m_1gh}$

3. (a) *A car of mass* **500kg** *is moving up a hill inclined at* $\sin^{-1}\left(\frac{1}{7}\right)$ *to the horizontal. The resistance to motion of the car is* **300N.** *If the power output of the car is* **84kW,** *find the acceleration of the car when its speed is* $35ms^{-1}$

(b) *A car of mass* **800kg** *is moving at a constant speed of* $20ms^{-1}$ *down a hill inclined at an angle* θ *to the horizontal. The resistance to motion of the car is* **1300N.** *If the power output of the car is* **10kW,** *show that* $\sin\theta = \frac{5}{49}$

4. *A light inelastic string is fixed at one end and passes under a moveable pulley* **A** *of mass* **4kg** *which hangs vertically. The other end of the string is attached to particle* **B** *of mass* **4kg** *which lies on a rough horizontal table. A second inelastic string connects* **B** *to a freely hanging particle* **C** *of mass* **10kg.** *The strings are passing over smooth fixed pulleys* **X** *and* **Y** *as shown*



If the system is released from rest and the coefficient of friction between **Q** *and the table is* **0·5,** *find the:*

(i) *accelerations of* **A, B** *and* **C**

(ii) *tension in the strings*

(iii) *reaction of pulley* **Y** *on the string*

5. *A particle moving with S·H·M has velocities of* $7\cdot 5ms^{-1}$ *and* $4ms^{-1}$ *as it passes through points* **P** *and* **Q** *which are* $0\cdot 9m$ *and* $0\cdot 2m$ *respectively from the end points of its path. Find the:*

(i) *length of its path and the period of the motion*

(ii) *maximum velocity and maximum acceleration*

(iii) *time it takes to travel directly from* **P** *to* **Q**

(iv) *time which elapses before it next passes through* **Q**

(v) *mean velocity during its motion from one extreme position to the other*

6. *A particle of mass* **m** *is suspended from a fixed point* **O** *by a light elastic string of natural length* **l**. *When the particle hangs in equilibrium, the extension of the string is* **d.** *The particle is then slightly vertically displaced from its equilibrium position and then released. Show that it moves with SHM of period* $2\pi\sqrt{\frac{d}{g}}$

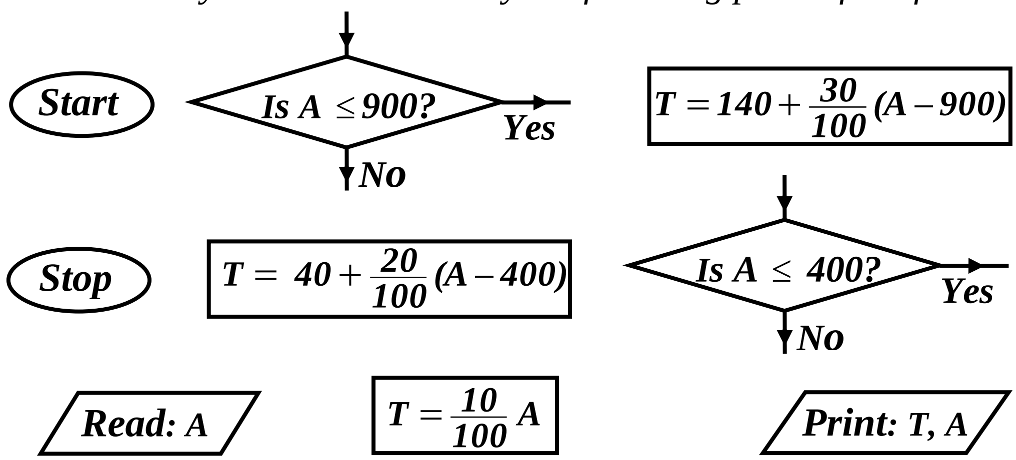
NUMERICAL METHODS

1. *Three decimal numbers* **X, Y** *and* **Z** *were rounded off to give* **x, y** *and* **z** *with errors* Δx, Δy *and* Δz *respectively. Show that the maximum relative error in the approximation of* $\frac{X}{Y-Z}$ *by* $\frac{x}{y-z}$ *is* $\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y-z}\right| + \left|\frac{\Delta z}{y-z}\right|$. *Hence find the absolute error and percentage error in* $\frac{1\cdot 6}{2\cdot 15 - 1\cdot 9}$ *and the interval within which its exact value is expected to lie*

2. *The income tax of an employee is calculated as follows:*

Taxable Income, A (£)	Tax rate (%)
01 – 400	10
401 – 900	20
Above 900	30

The taxation system is described by the following parts of the flowchart:



(i) *Arrange the given parts to form a complete logical flowchart*

(ii) *State the purpose of the flowchart*

(iii) *Performing a dry run for the flow chart and complete the table below*

A	T
1500	- - - -
750	- - - -

3. (i) *Show that the Newton Raphson formula for finding the natural logarithm of the* k^{th} *root of a number* **A** *is given by* $x_{n+1} = x_n - \frac{1}{k} + \frac{A}{k} e^{-kx_n}$

(ii) *Draw a flowchart that computes and prints the root in* **(a)** *above correct to* **3** *decimal places*

(iii) *Perform a dry run for your flowchart using* $x_o = 1 \cdot 25$, $A = 147$ *and* $k = 4$

4. *Find the percentage error in estimating* $\int_{0}^{\frac{\pi}{3}} \sec^2 x \, dx$ *using trapezium rule with six ordinates correct to* **4** *decimal places and state how it can be reduced*

5. *An equation has two iterative formulae* $x_{n+1} = 2x_n^2 e^{x_n}$ *and* $x_{n+1} = \frac{1}{2}e^{-x_n}$.

(i) *Use each formula twice to deduce with a reason the most suitable one when* $x_o = 0 \cdot 4$. *Hence state the root correct to* **3** *d.p*

(ii) *Without iterating deduce with a reason the most suitable formula when* $x_o = 0 \cdot 4$. *Hence use it twice to find the root correct to* **3** *d.p.*

(iii) *Show that the equation for the two iterative formulae is* $2xe^{x} - 1 = 0$

6.(a) *Show that the equation* $\cos(x^2) - x + 3 = 0$ *has a root between* **2·5** *and* **3.** *Hence use linear interpolation thrice to find the root correct to* **3** *d.p*

(b) *In a motor rally, car* **P** *was observed to be at distances of* **350m** *and* **400m** *from the starting line when the chasing car* **Q** *was at distances of* **240m** *and* **300m** *respectively. How far was car* **P** *from the starting line when car* **Q:**

(i) *started chasing it*

(ii) *caught up with it*

(c) *Use the fact that* $f(1{\cdot}15) = 1{\cdot}32$ *and* $f^{-1}(1 \cdot 26) = 1 \cdot 25$ *to find the value of* $f^{-1}(1 \cdot 22)$ *by linear interpolation correct to* **3 d.p**

7. *Locate the ranges where the two real roots of the equation* $x^4 - x - 10 = 0$ *lie. Hence use Newton Raphson method to find the least root correct to **3** d·p*

STATISTICS

1. *The table below shows the prices of three items for the years **2023** and **2024***

Item	PRICE (£)		Weights
	IN 2023	IN 2024	
A	150	153	5
B	250	261	2
C	525	588	3

*Taking **2023** as the base year, calculate the:*

(i) *simple aggregate price index for **2024.** Comment on your result*

(ii) *weighted mean price index for **2024.** Comment on your result*

(iii) *weighted aggregate price index for **2024.** Comment on your result*

(iv) *cost of items in **2023,** similar to the items in **2024** whose cost was **£540** using the result in **(iii)** above*

2. *The grades of **8** students in **UNEB, pre mock** and **post mock** were as follows:*

UNEB	B	A	O	C	B	E	O	D
Pre Mock	D	B	D	C	A	F	O	E
Post Mock	C	B	E	O	A	D	F	E

(a) *Calculate the rank correlation coefficient between the grades of:*

(i) UNEB *and* ***Pre Mock***

(ii) UNEB *and* ***Post Mock***

(b) *Which of the two mocks had a better correlation with **UNEB?** Give a reason*

3. *The table below shows the weights in kg of* ***100*** *babies:*

Weights	***2***	***2·5***	***4·5***	***6***	***7***	***8***
No of babies	***35***	***20***	***20***	***10***	***10***	***5***

(i) *Calculate the mean, variance and median for the above data*

(ii) *Assuming this was a sample taken from a normal population, find the* ***90%*** *confidence interval for the mean weight of all babies*

4. *The table below shows the weights in kg of* ***40*** *boys:*

Weights	***30 – < 35***	***35 – < 40***	***40 – < 55***	***55 – < 60***	***60 – < 65***
Frequency	***8***	***5***	***12***	***9***	***6***

(a) *Calculate the mean, mode and percentage of boys heavier than* ***45 kg***

(b) *Draw an ogive for the data and use it to estimate the:*

(i) *median weight*

(ii) *quartile deviation*

(iii) *range of the weights of the middle* ***70%*** *of the boys*

(c) *Draw a histogram for the data and use it to estimate the modal weight*

5. *The price index of an item in* ***2022*** *based on* ***2023*** *was* ***88.*** *Its price index in* ***2024*** *based on* ***2023*** *was* ***132.*** *Find its:*

(i) *price index in* ***2024*** *based on* ***2022***

(ii) *price in* ***2022*** *if its price in* ***2024*** *was* ***£600***

PROBABILITY

1. *A box contains* **150** *red and* **50** *blue pens.* **48** *pens are drawn in succession at random from the box with replacement. Find the probability of picking:*

(i) *exactly* **30** *red pens*

(ii) *at least* **29** *red pens*

(iii) *at least* **8** *but less than* **20** *blue pens*

2. *A continuous* $\boldsymbol{r \cdot v}$ $X \sim R(3, 15)$.

(a) *Write down the p·d·f of* **X** *and sketch it*

(b) *Find:* **(i)** $E(X)$ **(ii)** $Var(X)$ **(iii)** *the upper quartile of* **X**

(iv) $P(4 < X < 10)$ **(v)** $P(|X - 7| < 2 / X \geq 6)$

(vi) *the distribution function of* **X** *and sketch it*

3. *Box* **A** *contains* **4** *red and* **3** *green pens, box* **B** *contains* **3** *red and* **4** *green pens, while box* **C** *contains* **5** *red and* **2** *green pens. Boxes* **A, B** *and* **C** *are in the ratio* **2 : 3 : 5** *respectively as likely to be picked. If a box is selected at random and two pens are picked from it without replacement,*

(a) *find the probability of picking:*

(i) *pens of different colours*

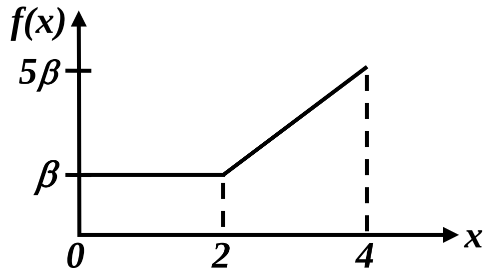
(ii) *box* **B** *given that the pens drawn are of the same colour*

(b) *If* **X** *is the number of red pens drawn, find the:*

(i) *probability distribution of* **X**

(ii) *median, mean and variance of* **X**

4. *The p·d·f of a continuous* **r·v X** *is distributed as follows:*



Find:

(i) *the value of* β

(ii) *the equations of the p·d·f*

(iii) *the mean and median of* **X**

(iv) *the cumulative distribution function of* **X** *and sketch it*

(v) $P(X > 1 / X < 3)$

5. *A continuous r·v* **X** *has the following distribution function:*

$$F(x) = \begin{cases} 0 & , \quad x \le 0 \\ \dfrac{\sqrt{2}}{2}\sin x & , \quad 0 \le x \le \lambda \\ 1 - \dfrac{\sqrt{2}}{2}\cos x & , \quad \lambda \le x \le \dfrac{\pi}{2} \\ 1 & , \quad x \ge \dfrac{\pi}{2} \end{cases}$$

(a) *Show that* $\lambda = \dfrac{\pi}{4}$

(b) *Find:*

(i) $P\left(\left|X - \dfrac{\pi}{4}\right| \le \dfrac{\pi}{12}\right)$

(ii) *the equations of the p·d·f and sketch it, hence deduce the mean of* **X**

(iii) $E\left(3X - \dfrac{\pi}{3}\right)$

6. *A discrete* $\boldsymbol{r \cdot v}$ $\boldsymbol{X}$ *has the following* $p \cdot d \cdot f$**:**

$$P(X = x) = \begin{cases} \frac{1}{60}(ax + b) & , \quad x = 1, 2, 3, 4 \\ 0 & , \quad otherwise \end{cases}$$

Given that $F(3) = \frac{13}{20}$, *find the:*

(i) *values of* **a** *and* **b,** *hence sketch the* $p \cdot d \cdot f$ *of* **X**

(ii) $P(X > 1 / X < 3)$

(iii) *mean and variance of* **X**

(iv) $E(3X - 4)$ *and* $Var(3X - 4)$

7. (a) *A random variable* **X** *is binomially distributed with mean* **4·8** *and variance* **2·88.** *Find* $P(X < 6)$

(b) *A student answers* **12** *questions. The chance of passing each question is* $\frac{1}{3}$.

Find the probability of passing:

(i) *exactly* **7** *questions*

(ii) *at least* **2** *questions*

8. *A random variable* **X** *is normally distributed such that* $P(X < 76) = 0 \cdot 9772$ *and* $P(72 < X < 76) = 0 \cdot 044$. *Find:*

(i) *the mean and standard deviation of* **X**

(ii) $P(X > 45)$

(iii) *the interval which contains the middle* **95%** *of distribution*

9. *A random sample of* **100** *nails taken from a normal population had the following lengths* x *in cm:* $\sum x = 380$ *and* $\sum x^2 = 1840$. *Find the:*

(i) *unbiased estimate for the population variance*

(ii) $90{\cdot}8\%$ *confidence interval for the population mean*

10. *A random sample of* **36** *items drawn from a normal population is such that the* **95%** *confidence interval for the mean of all the items is* $[67{\cdot}9,\ 77{\cdot}7]$. *Find the* **90%** *confidence limits for the mean of all the items*

11. *Given that* $P(A \cup B) = \frac{9}{10}$, $P(A/B) = \frac{1}{3}$ *and* $P(B/A) = \frac{2}{5}$, *find:*

(i) $P(A)$

(ii) $P(\overline{A}/\overline{B})$

(iii) *P(A or B but not both A and B)*

12. *At a certain party,* **25%** *of the guests are women. Nile beer and Bell beer are the only drinks available for the guests.* **40%** *of the women and* **70%** *of the men take Nile beer. Of the men taking Nile beer,* **80%** *got drunk and of the men taking Bell beer,* **60%** *got drunk. Of the women taking Nile beer,* **50%** *got drunk and of the women taking Bell beer,* **40%** *got drunk. Find the probability that a randomly selected guest:*

(i) *takes Nile beer*

(ii) *got drunk*

(iii) *got drunk given that is a woman*

(iv) *is a man given that he got drunk for taking Nile beer*

13. ***(a)*** *Events* ***A*** *and* ***B*** *are such that* $P(A)=\frac{2}{3}$, $P(B)=\frac{1}{4}$ *and* $P(A\cup B)=\frac{17}{24}$.

Find: ***(i)*** $P(A\cap B)$ ***(ii)*** $P(\overline{A}\cap B)$ ***(iii)*** $P(\overline{A}\cap\overline{B})$ ***(iv)*** $P(\overline{A}\cup\overline{B})$

(b) *If* $\overline{A}$ *and* $\overline{B}$ *are independent events,*

(i) *Show that the events* ***A*** *and* ***B*** *are also independent*

(ii) *find* $P(B)$ *and* $P(\overline{A}\cup\overline{B})$, *if* $P(A)=0\cdot 375$ *and* $P(A\cup B)=0\cdot 75$

(c) *Find the possible values of* $P(A)$, *if* ***A*** *and* ***B*** *are independent events such that* $P(A\cap B)=0\cdot 3$ *and* $P(A\cup B)=0\cdot 875$

14. *A task in mathematics is given to three students whose chances of solving it are* $\frac{1}{3}$, $\frac{1}{4}$ *and* $\frac{1}{5}$ *respectively. Find the probability that:* ***(i)*** *the task is solved*

(ii) *only one student solves it* ***(iii)*** *at least two of them solved it*

15. *Two soldiers* ***A*** *and* ***B*** *in that order take turns shooting a bullet at a target. The first one to hit the target wins the game. If their chances of hitting the target on each occasion they shoot are* $\frac{1}{3}$ *and* $\frac{1}{4}$ *respectively, find the chance that:*

(i) A *wins the game on his third shot* ***(ii) A*** *wins the game.*

16. *Mutually exclusive events* ***A*** *and* ***B*** *are such that* $P(A\cup B)=0\cdot 75$ *and* $P(A)=0\cdot 27$, *find:* ***(i)*** $P(\overline{A}\cup B)$ ***(ii)*** $P(\overline{A}\cap\overline{B})$ ***(iii)*** $P(A\cap B)'$

17. *Exhaustive events* ***A*** *and* ***B*** *are such that* $5P(A)=4P(B)$ *and* $P(A\cap B)=\frac{1}{5}$.

Find: ***(i)*** $P(A)$ ***(ii)*** $P(A/B)$

END

STATISTICS AND PROBABILITY:

1a) The probability that Amos goes to school on cloudy day is $\frac{7}{10}$ and $\frac{1}{5}$ on a clear day. If the probability of a cloudy day is $\frac{3}{5}$, find the probability that the day was cloudy given that he did not go to school.

b) If P and Q are independent event;

i) Show that P' and Q are also independent.

ii) Find $P(Q)$ given that $P(P)=0.25$ and $P(P \cup Q)=0.75$.

2. During the national teachers' conference, 40% of the delegates supported the idea of the 10% salary increase for the financial year 2019. Given that journalists randomly interviewed 450 delegates, find the probability that;

   a) Less than 150 delegates supported the salary increase,

   b) Between 160 and 170 delegates inclusive supported the salary increase.

3. The table below shows the heights measured in cm for a group of senior six students;

Height	177-186	187-191	192-196	197-201	202-206	207-216
Frequency	12	8	8	9	7	6

   a) Draw a histogram. Hence state the modal class.

   b) Calculate the (i) mean (ii) standard deviation. (ii) Mode

4. Events A, b and C are such that $P(A)=x$, $P(B)=y$ and $P(C)=x+y$. If $P(A \cup N)=0.6$ and $P\left(B/A\right)=0.2$,

   (i) Show that $4x+5y=3$.

   (ii) Given that B and C are mutually exclusive and that $P(B \cup C)=0.9$, determine another equation in x and y.

   (iii) Hence find the values of x and y.

5. Of the 30 drivers interviewed, 9 have been involved in a car accident at some time. Of those who have been involved in the accident, 5 wear glasses. The probability of wearing glasses, given that the driver has not had a car accident is $\frac{1}{3}$. Find the probability that a;

   (a) Person chosen at random wears glasses.

   (b) Glasses wearer has been an accident victim.

6a) Given that A and B are two events such that $P(A')=0.3$, $P(B)=0.1$ *and* $P\left(A/B\right)=0.2$. Find (i) $P(A \cup B)$ (ii) $P\left(A/B'\right)$

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b) A box A contains 1 red, 3 green and 1 blue ball. Box B contains 2 red, 1 green and 2 blue balls. A balanced die is thrown and if the throw is a six, box A is chosen, otherwise box B is chosen. A ball is drawn at random from the chosen box. Given that a green ball is drawn, what is the probability that it came from box A?

7. The cumulative distribution function of a discrete random variable x is as shown in the table below.

x	1	2	3	4
$F(x)$	0. 14	0. 47	0. 79	1. 00

Find the (i) $P(2 < x \leq 4)$ (ii) median of x (iii) $P\left(x < 3 / 2 \leq x < 4\right)$ (iv) mean of x.

8. The probability density function of a continuous random variable x is given by;

$$f(x) = \begin{cases} \frac{2}{13}(x+1), & 0 \leq x \leq a \\ \frac{2}{13}(5-x), & a \leq x \leq b \\ 0 & elsewhere \end{cases},$$ find the values of a and b. Hence, sketch $f(x)$.

(i) Calculate the mean and standard deviation of x.

(iii) Find $P(x < 2.5)$.

9. The Maths and Physics examination marks of a certain school are given in the following table.

Maths (x)	28	34	36	42	52	54	60
Physics(y)	54	62	68	70	76	68	74

a)i) Plot the marks on the scatter diagram and comment on the relationship between the two subjects.

ii) Draw a line of best fit and use it to predict the Physics mark of a student whose Maths mark is 50.

b) Calculate the rank correlation coefficient between the marks. Comment on the significance of Maths on Physics performance based on 5% level of significance

10a) A random sample of ten packets is taken. These have masses (Measured in kg) of x_1, x_2, ..., x_{10} such that; $\sum_{i=1}^{10} xi = 2.57$ and $\sum_{i=1}^{10} x^2 i = 0.6610$. Calculate a 95% confidence limits for the mean.

b) The weights of ball bearings are normally disturbed with mean 25gram and standard demotion 4 grams. If a random sample of 16 ball bearings is taken, find the probability that the mean of the sample his between 24.12 grams and 26.73 grams.

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11. The table below the number of reported accidents to workers in a certain factory over the past 12 years.

a) Find the mean and median of the workers

Age of Worker	15-	20-	25-	30-	35-	40- < 45
Number of employees	42	52	28	20	18	16

b) Plot a cumulative frequency curve and use it to estimate the;

(i) Proportion of employees who are not more than 27 years old.
(ii) 80% central limits of the age of employees.

12. A survey of the mass (kg) of girls in the certain school in the final year was taken and the results are shown below;

Mass (kg)	< 45	<50	<60	<75	<80	<85	<90	<100	<105	<110
Cumulative frequency	0	9	25	46	63	77	82	89	93	94

(a) Display the information on a histogram and hence find the modal mass

(b) Calculate the standard deviation of the mass.

13. A random variable X has the probability density function

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & elsewhere \end{cases}$$

a) Show that the variance of x is $\dfrac{(b-a)^2}{12}$.

b) If the expected value and variance of the distribution is 1 and $\dfrac{3}{4}$ respectively,

(i) $P(x < 0)$

(ii) Value of a such that $P(X > a + \sigma) = \dfrac{1}{4}$. Where σ is the standard deviation of X.

14. A pharmacist had the following records of unit price and quantities of drugs sold for the years 2010 and 2011.

Drug	Unit price per carton		Quantities per carton	
	2010	**2011**	**2010**	**2011**
Aspirin	80	125	40	45
Panadol	100	90	70	90
Quinine	55	75	8	10
Coartem	90	100	10	10

Taking 2010 as the base year,

(i) Calculate the price relatives for each drug in 2011.
(ii) Obtain the simple aggregate price index number for 2011.

(iii) Calculate the weighted price index and comment on it.
(iv) Calculate the weighted aggregate price index and comment on it.

MECHANICS:

15. A body of mass 0.2kg is acted upon by a force $\mathbf{F} = 8t\,\mathbf{i} - 4t^2\mathbf{j} + 2(3 - t^2)\mathbf{k}\ N$. Initially, the body is at rest at a point with position vector $-10\,\mathbf{i} + 12\mathbf{j} - 4\,\mathbf{k}$. Find the
(i) velocity after time t seconds,
(ii) Distance covered by the body in 2seconds.

16. An object of mass 4kg moves under the action of three forces $\mathbf{F}_1 = \begin{pmatrix} \frac{t}{2} - 1 \\ t - 3 \end{pmatrix} N$,

$\mathbf{F}_2 = \begin{pmatrix} \frac{t}{2} + 2 \\ \frac{t}{2} - 4 \end{pmatrix} N$ and $\mathbf{F}_3 = \begin{pmatrix} t - 3 \\ 3t - 2 \end{pmatrix} N$ at time t seconds. Find the
(i) Work done by the resultant force when the object is displaced from point $A(4,\ \ 5)$ to a point $B\,(7,\ -9)$ within a time of $1\,s$.
(ii) Size of the acceleration of the object after a time of $2\ s$.
(iii) Power developed when the object moves with velocity of $3\mathbf{i} - 6\mathbf{j}\ m\,s^{-1}$ after a time of $2\ s$.

17. Two smooth inclined planes meet at right angles, the inclination of one to the horizontal being 30° and the other being 60°. Bodies of masses 2kg and 4kg lie on the respective planes, and the two masses are joined by a light inextensible string passing over a smooth fixed pulley at the intersection of the planes.
(i) If the masses are released from rest, find the acceleration of the masses and tension of the string.
(ii) If the surface with 4kg mass experiences a frictional force of 0.25N, find the new acceleration of the masses when the system is set from rest.

18. Four forces represented by $i - 4j, 3i + 6j, -9i + j\ and\ 5i - 3j$, at a points with position vectors $3i - j, 2i + 2j, -i - j\ and - 3i + 4j$ respectively.

(a) Show that the forces reduce to a couple and find its moment
(b) If the fourth force is removed, find the magnitude and direction of the resultant forces.

19a) If the horizontal range of a particle projected with a velocity v is a, show that the greatest height x, attained is given by the equation $16gx^2 - 8v^2x + g\,a^2 = 0$, where g is acceleration due to gravity.

b) A small ball is projected from a point A on a level ground and passes at a height of 2metres over a point on the ground at a distance of 12m from A. The ball is caught at a height of 1metre by a girl who is 24metres from A. Find the angle of projection of the ball as it leaves point A.(g=10m/s^2)

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20a) A particle of mass 2kg is pushed up a rough inclined plane whose slope is 3 in 5 by a horizontal force of 46.25N. The particle starts from rest at P and is moved up a line of greatest slope of the plane to the point Q, where PQ=15m. The coefficient of friction between the plane and the particle is 0.25. Find the speed acquired by the particle at Q (Take g=$10ms^{-2}$)

b) A pump draws water from a tank and delivers to a height of 10m at the end of a horse. The cross-sectional area of the horse is $10cm^2$ and the water leaves the end of the horse with a speed of $7.5\,m\,s^{-1}$. Find the rate at which the pump is working.

21a) A particle is projected at an angle of 30^0 with a speed of $21\,m\,s^{-1}$. If the point of projection is 5m above the horizontal grounds, find the horizontal distance that the particle travels before striking the ground.

b) A boy throws a ball at an initial speed of 40 ms^{-1} at an angle of elevation α. Show that the times of flight corresponding to the horizontal range of 8 m are positive roots of the equation $T^4 - 64T^2 + 256 = 0$. (Take g=10 ms^{-2}).

22. A light inextensible string has one end attached to a ceiling. The string passes under a smooth moveable pulley of mass 2 kg and then over a smooth fixed pulley. Particle of mass 5 kg is attached at the free end of the string. The sections of the string not in contact with the pulleys are vertical. If the system is released from rest and moves in a vertical plane, find the:

i) Acceleration of the system.

ii) Tension in the string.

(iii) Distance moved by the moveable pulley in 1.5 s.

23a) A body of mass 5 kg slides a distance of 8 m down a rough plane inclined at an angle of $\sin^{-1}\left(\frac{4}{5}\right)$to the horizontal. If the coefficient of friction is 0.4, find the velocity attained by the body.

b) A particle of mass 50 kg is suspended by two light inelastic strings of lengths 9 m and 12m attached to two points distant 15 m apart. Calculate the tensions in the strings.

24. A rod AB 1m long has a weight of 20N acting at a point 60cm from A. It rests horizontally with A hinged on a vertical wall. A string BC is fastened to the wall at C, 75cm vertically about A find the;

(i) Tension in the string

(ii) Reaction from at point A.

25a) A particle travelling in a straight line with the constant acceleration covers distances x and y in the 3^{rd} and 4^{th} seconds of its motion respectively. Show that its initial speed is given by U = $\frac{1}{2}(7x - 5y)$.

b) Two stations A and B are a distance of $6x$m apart along a straight path. A train starts from rest at A and accelerates uniformly until it attains a speed of Vms^{-1} covering a distance of x m. The train then maintains this speed until it has travelled a further $3x$ m . It then retards uniformly to rest at B. show that if T is the time taken for a train to travel from A to B then T = $(\frac{9x}{V})$ s

26. Forces of magnitudes **6N, 6N, 4N, 10N** and **8N** act along **AB, BC, CD, DB** and **AD** respectively, in the directions indicated by the order of the letters of a rectangle **ABCD** of dimensions 4m by 3m. Find the;

(i) Magnitude of their resultant
(ii) Equation of the line of action of their resultant force
(iii) Distance from **B** where it cuts **AB.**

27a) A,B,C and D are the points (0,0),(10,0),(7,4) and (3,4) respectively. If AB, BC, CD and DA are made of a thin wire of uniform mass. Find the coordinates of the centre of gravity.

b)i) If instead ABCD is a uniform lamina, find its centre of gravity G

ii) If the lamina hung from B, find the angle AB makes with the vertical

28. An object of mass 5kg is initially at rest at a point whose position vector is -2i +j. If it is acted upon by a force, F = 2i +3j -4k, find

a) The acceleration

b) The velocity after 3s

c) Its distance from the origin after 3s.

29a) A car is being driven at $20\ m\ s^{-1}$ on a bearing of 040°. the wind is blowing from 330° with a speed of $10\ m\ s^{-1}$. Find the velocity of the wind as experienced by the driver of the car.

b) At 8$a.m$ two boats A and B are 5.2km apart with A due west of B, and B travelling in a direction N20°w at a steady speed of 13kmh^{-1}. If A travels due north at 12kmh^{-1}, determine;

(i) The path of B relative to A

(ii) The distance between A and B at 8:30am.

30. A particle of mass M kg is attached to a string of natural length L and modulus of elasticity 4Mg. The other end is attached to a fixed point O. If the particle is released from rest at O.

(i) Find the greatest distance below O reached by the particle and its acceleration at this point.

(ii) Show that the speed of the particle is $1.5\sqrt{gL}$ when the acceleration is instantaneously zero.

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NUMERICAL METHODS:

31a)The table below shows x and f(x);

x	50.24	48.11	46.93	44.06
f(x)	4.116	7.621	9.043	11.163

Use linear interpolation or extrapolation to estimate;
(i) x when f(x)=8.614
(ii) f^{-1} (5 1.07)

b) Use trapezium rule with six ordinates to find the value of $\int_1^2 \frac{x^2}{x^2+1}\,dx$, correct to three decimal places.

c) Find the exact value of $\int_1^2 \frac{x^2}{x^2+1}\,dx$, correct to three decimal places. How can you improve on the degree of accuracy?

32a) Use the trapezium rule with 6 strips to evaluate $\int_1^2 (x-1)\, In\ x\, dx$ correct to 4 decimal places.

b) Find the exact value of $\int_1^2 (x-1)\, In\ x\, dx$, Hence find the maximum possible error in your calculations in (a) above.

33. If the numbers x and y are approximations with errors Δx and Δy respectively. Show that the maximum absolute error in the approximations of x^2y is given by

$|\boldsymbol{2xy\Delta x}| + |\boldsymbol{x^2\Delta y}|$ Hence find the limits within which the true value of x^2y lies given that $x = 2.8 \pm 0.016$ and $y = 1.44 \pm 0.008$

34a) Show that the equation $3x^2 - x - 5 = 0$ has a real root between x= -1.4 and x= -1.2.
b) Use linear interpolation to estimate the initial root
c) Using the root in (b), use Newton – Raphson Method to find the root, correct to two decimal places.

35a) The dimensions of a rectangle are 8cm and 4.26.
(i) State the maximum possible error in each dimension.
(ii) Find the range within which the area of the rectangle lies. (correct to 2 decimal places)
b) The radius r and height h of a cylinder are measured with corresponding errors Δr and Δh respectively. Show that the maximum possible error in the volume is $\left|\frac{\Delta h}{h}\right| + 2\left|\frac{\Delta r}{r}\right|$

36a) Derive the simplest formula based on Newton Raphson's method to show that for the equation $3x = 1n3$ it satisfies $x_{r+1} = \frac{1}{3}\left\{\frac{e^{3x_r}(3x_r - 1) + 3}{e^{3x_r}}\right\}$

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b) Draw a flow chart that;

i) reads the initial approximation x_0,

ii) Accepts estimations to 4 significant figures

iii) Prints the root to $4sf$.

iv) Perform a dry-run for your flow chart for $x_0 = \frac{1}{3}$.

37a) Show that the iterative formula based on Newton Raphson's Method for approximating the root of the equation $2\ln x = x - 1$ is given by;

$$x_{n+1} = x_n \left(\frac{2\ln x_n - 1}{x_n - 2}\right) \text{ n = 0, 1, 2. . . . .}$$

(b) Draw a flow chart that,

(i) Reads the initial approximation x_o of the root.

(ii) Computes and prints the root correct to two decimal places, using the formula in (a) above.

END

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STATISTICS AND PROBABILITY:

1a) The probability that Amos goes to school on cloudy day is $\frac{7}{10}$ and $\frac{1}{5}$ on a clear day. If the probability of a cloudy day is $\frac{3}{5}$, find the probability that the day was cloudy given that he did not go to school.

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   a) Less than 150 delegates supported the salary increase,

   b) Between 160 and 170 delegates inclusive supported the salary increase.

3. The table below shows the heights measured in cm for a group of senior six students;

Height	177-186	187-191	192-196	197-201	202-206	207-216
Frequency	12	8	8	9	7	6

   a) Draw a histogram. Hence state the modal class.

   b) Calculate the (i) mean (ii) standard deviation. (ii) Mode

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   (i) Show that $4x+5y=3$.

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5. Of the 30 drivers interviewed, 9 have been involved in a car accident at some time. Of those who have been involved in the accident, 5 wear glasses. The probability of wearing glasses, given that the driver has not had a car accident is $\frac{1}{3}$. Find the probability that a;

   (a) Person chosen at random wears glasses.

   (b) Glasses wearer has been an accident victim.

6a) Given that A and B are two events such that $P(A')=0.3$, $P(B)=0.1$ *and* $P\left(A/B\right)=0.2$. Find (i) $P(A \cup B)$ (ii) $P\left(A/B'\right)$

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b) A box A contains 1 red, 3 green and 1 blue ball. Box B contains 2 red, 1 green and 2 blue balls. A balanced die is thrown and if the throw is a six, box A is chosen, otherwise box B is chosen. A ball is drawn at random from the chosen box. Given that a green ball is drawn, what is the probability that it came from box A?

7. The cumulative distribution function of a discrete random variable x is as shown in the table below.

x	1	2	3	4
$F(x)$	0. 14	0. 47	0. 79	1. 00

Find the (i) $P(2 < x \leq 4)$ (ii) median of x (iii) $P\left(x < 3 / 2 \leq x < 4\right)$ (iv) mean of x.

8. The probability density function of a continuous random variable x is given by;

$$f(x) = \begin{cases} \frac{2}{13}(x+1), & 0 \leq x \leq a \\ \frac{2}{13}(5-x), & a \leq x \leq b \\ 0 & elsewhere \end{cases},$$ find the values of a and b. Hence, sketch $f(x)$.

(i) Calculate the mean and standard deviation of x.

(iii) Find $P(x < 2.5)$.

9. The Maths and Physics examination marks of a certain school are given in the following table.

Maths (x)	28	34	36	42	52	54	60
Physics(y)	54	62	68	70	76	68	74

a)i) Plot the marks on the scatter diagram and comment on the relationship between the two subjects.

ii) Draw a line of best fit and use it to predict the Physics mark of a student whose Maths mark is 50.

b) Calculate the rank correlation coefficient between the marks. Comment on the significance of Maths on Physics performance based on 5% level of significance

10a) A random sample of ten packets is taken. These have masses (Measured in kg) of x_1, x_2, ..., x_{10} such that; $\sum_{i=1}^{10} xi = 2.57$ and $\sum_{i=1}^{10} x^2 i = 0.6610$. Calculate a 95% confidence limits for the mean.

b) The weights of ball bearings are normally disturbed with mean 25gram and standard demotion 4 grams. If a random sample of 16 ball bearings is taken, find the probability that the mean of the sample his between 24.12 grams and 26.73 grams.

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11. The table below the number of reported accidents to workers in a certain factory over the past 12 years.

Age of Worker	15-	20-	25-	30-	35-	40- < 45
Number of employees	42	52	28	20	18	16

a) Find the mean and median of the workers

b) Plot a cumulative frequency curve and use it to estimate the;

(i) Proportion of employees who are not more than 27 years old.
(ii) 80% central limits of the age of employees.

12. A survey of the mass (kg) of girls in the certain school in the final year was taken and the results are shown below;

Mass (kg)	< 45	<50	<60	<75	<80	<85	<90	<100	<105	<110
Cumulative frequency	0	9	25	46	63	77	82	89	93	94

(a) Display the information on a histogram and hence find the modal mass

(b) Calculate the standard deviation of the mass.

13. A random variable X has the probability density function

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & elsewhere \end{cases}$$

a) Show that the variance of x is $\dfrac{(b-a)^2}{12}$.

b) If the expected value and variance of the distribution is 1 and $\dfrac{3}{4}$ respectively,

(i) $P(x < 0)$

(ii) Value of a such that $P(X > a + \sigma) = \dfrac{1}{4}$. Where σ is the standard deviation of X.

14. A pharmacist had the following records of unit price and quantities of drugs sold for the years 2010 and 2011.

Drug	Unit price per carton		Quantities per carton	
	2010	**2011**	**2010**	**2011**
Aspirin	80	125	40	45
Panadol	100	90	70	90
Quinine	55	75	8	10
Coartem	90	100	10	10

Taking 2010 as the base year,

(i) Calculate the price relatives for each drug in 2011.
(ii) Obtain the simple aggregate price index number for 2011.

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(iii) Calculate the weighted price index and comment on it.
(iv) Calculate the weighted aggregate price index and comment on it.

MECHANICS:

15. A body of mass 0.2kg is acted upon by a force $\mathbf{F} = 8t\,\mathbf{i} - 4t^2\mathbf{j} + 2(3 - t^2)\mathbf{k}\ N$. Initially, the body is at rest at a point with position vector $-10\,\mathbf{i} + 12\mathbf{j} - 4\,\mathbf{k}$. Find the
(i) velocity after time t seconds,
(ii) Distance covered by the body in 2seconds.

16. An object of mass 4kg moves under the action of three forces $\mathbf{F}_1 = \begin{pmatrix} \frac{t}{2} - 1 \\ t - 3 \end{pmatrix} N$,

$\mathbf{F}_2 = \begin{pmatrix} \frac{t}{2} + 2 \\ \frac{t}{2} - 4 \end{pmatrix} N$ and $\mathbf{F}_3 = \begin{pmatrix} t - 3 \\ 3t - 2 \end{pmatrix} N$ at time t seconds. Find the
(i) Work done by the resultant force when the object is displaced from point $A(4,\ 5)$ to a point $B(7,\ -9)$ within a time of $1\,s$.
(ii) Size of the acceleration of the object after a time of $2\ s$.
(iii) Power developed when the object moves with velocity of $3\mathbf{i} - 6\mathbf{j}\ m\,s^{-1}$ after a time of $2\ s$.

17. Two smooth inclined planes meet at right angles, the inclination of one to the horizontal being 30° and the other being 60°. Bodies of masses 2kg and 4kg lie on the respective planes, and the two masses are joined by a light inextensible string passing over a smooth fixed pulley at the intersection of the planes.
(i) If the masses are released from rest, find the acceleration of the masses and tension of the string.
(ii) If the surface with 4kg mass experiences a frictional force of 0.25N, find the new acceleration of the masses when the system is set from rest.

18. Four forces represented by $i - 4j, 3i + 6j, -9i + j$ *and* $5i - 3j$, at a points with position vectors $3i - j, 2i + 2j, -i - j$ *and* $-3i + 4j$ respectively.

(a) Show that the forces reduce to a couple and find its moment
(b) If the fourth force is removed, find the magnitude and direction of the resultant forces.

19a) If the horizontal range of a particle projected with a velocity v is a, show that the greatest height x, attained is given by the equation $16gx^2 - 8v^2x + ga^2 = 0$, where g is acceleration due to gravity.

b) A small ball is projected from a point A on a level ground and passes at a height of 2metres over a point on the ground at a distance of 12m from A. The ball is caught at a height of 1metre by a girl who is 24metres from A. Find the angle of projection of the ball as it leaves point A.($g=10m/s^2$)

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20a) A particle of mass 2kg is pushed up a rough inclined plane whose slope is 3 in 5 by a horizontal force of 46.25N. The particle starts from rest at P and is moved up a line of greatest slope of the plane to the point Q, where PQ=15m. The coefficient of friction between the plane and the particle is 0.25. Find the speed acquired by the particle at Q (Take g=10ms^{-2})

b) A pump draws water from a tank and delivers to a height of 10m at the end of a horse. The cross-sectional area of the horse is 10cm^2 and the water leaves the end of the horse with a speed of $7.5\,m\,s^{-1}$. Find the rate at which the pump is working.

21a) A particle is projected at an angle of 30^0 with a speed of $21\,m\,s^{-1}$. If the point of projection is 5m above the horizontal grounds, find the horizontal distance that the particle travels before striking the ground.

b) A boy throws a ball at an initial speed of 40 ms^{-1} at an angle of elevation α. Show that the times of flight corresponding to the horizontal range of 8 m are positive roots of the equation $T^4 - 64T^2 + 256 = 0$. (Take g=10 ms^{-2}).

22. A light inextensible string has one end attached to a ceiling. The string passes under a smooth moveable pulley of mass 2 kg and then over a smooth fixed pulley. Particle of mass 5 kg is attached at the free end of the string. The sections of the string not in contact with the pulleys are vertical. If the system is released from rest and moves in a vertical plane, find the:

i) Acceleration of the system.

ii) Tension in the string.

(iii) Distance moved by the moveable pulley in 1.5 s.

23a) A body of mass 5 kg slides a distance of 8 m down a rough plane inclined at an angle of $\sin^{-1}\left(\frac{4}{5}\right)$to the horizontal. If the coefficient of friction is 0.4, find the velocity attained by the body.

b) A particle of mass 50 kg is suspended by two light inelastic strings of lengths 9 m and 12m attached to two points distant 15 m apart. Calculate the tensions in the strings.

24. A rod AB 1m long has a weight of 20N acting at a point 60cm from A. It rests horizontally with A hinged on a vertical wall. A string BC is fastened to the wall at C, 75cm vertically about A find the;

(i) Tension in the string

(ii) Reaction from at point A.

25a) A particle travelling in a straight line with the constant acceleration covers distances x and y in the 3rd and 4th seconds of its motion respectively. Show that its initial speed is given by U = $\frac{1}{2}(7x - 5y)$.

b) Two stations A and B are a distance of $6x$m apart along a straight path. A train starts from rest at A and accelerates uniformly until it attains a speed of Vms^{-1} covering a distance of x m. The train then maintains this speed until it has travelled a further $3x$ m . It then retards uniformly to rest at B. show that if T is the time taken for a train to travel from A to B then T = $(\frac{9x}{V})$ s

26. Forces of magnitudes **6N, 6N, 4N, 10N** and **8N** act along **AB, BC, CD, DB** and **AD** respectively, in the directions indicated by the order of the letters of a rectangle **ABCD** of dimensions 4m by 3m. Find the;

(i) Magnitude of their resultant
(ii) Equation of the line of action of their resultant force
(iii) Distance from **B** where it cuts **AB.**

27a) A,B,C and D are the points (0,0),(10,0),(7,4) and (3,4) respectively. If AB, BC, CD and DA are made of a thin wire of uniform mass. Find the coordinates of the centre of gravity.

b)i) If instead ABCD is a uniform lamina, find its centre of gravity G

ii) If the lamina hung from B, find the angle AB makes with the vertical

28. An object of mass 5kg is initially at rest at a point whose position vector is -2i +j. If it is acted upon by a force, F = 2i +3j -4k, find

a) The acceleration

b) The velocity after 3s

c) Its distance from the origin after 3s.

29a) A car is being driven at $20\ m\ s^{-1}$ on a bearing of 040°. the wind is blowing from 330° with a speed of $10\ m\ s^{-1}$. Find the velocity of the wind as experienced by the driver of the car.

b) At 8$a.m$ two boats A and B are 5.2km apart with A due west of B, and B travelling in a direction N20°w at a steady speed of 13kmh^{-1}. If A travels due north at 12kmh^{-1}, determine;

(i) The path of B relative to A

(ii) The distance between A and B at 8:30am.

30. A particle of mass M kg is attached to a string of natural length L and modulus of elasticity 4Mg. The other end is attached to a fixed point O. If the particle is released from rest at O.

(i) Find the greatest distance below O reached by the particle and its acceleration at this point.

(ii) Show that the speed of the particle is $1.5\sqrt{gL}$ when the acceleration is instantaneously zero.

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NUMERICAL METHODS:

31a)The table below shows x and f(x);

x	50.24	48.11	46.93	44.06
f(x)	4.116	7.621	9.043	11.163

Use linear interpolation or extrapolation to estimate;
(i) x when f(x)=8.614
(ii) f^{-1} (5 1.07)

b) Use trapezium rule with six ordinates to find the value of $\int_1^2 \frac{x^2}{x^2+1}\,dx$, correct to three decimal places.

c) Find the exact value of $\int_1^2 \frac{x^2}{x^2+1}\,dx$, correct to three decimal places. How can you improve on the degree of accuracy?

32a) Use the trapezium rule with 6 strips to evaluate $\int_1^2 (x-1)\,In\;x\,dx$ correct to 4 decimal places.

b) Find the exact value of $\int_1^2 (x-1)\,In\;x\,dx$, Hence find the maximum possible error in your calculations in (a) above.

33. If the numbers x and y are approximations with errors Δx and Δy respectively. Show that the maximum absolute error in the approximations of x^2y is given by

$|\boldsymbol{2xy\Delta x}| + |\boldsymbol{x^2\Delta y}|$ Hence find the limits within which the true value of x^2y lies given that $x = 2.8 \pm 0.016$ and $y = 1.44 \pm 0.008$

34a) Show that the equation $3x^2 - x - 5 = 0$ has a real root between x= -1.4 and x= -1.2.
b) Use linear interpolation to estimate the initial root
c) Using the root in (b), use Newton – Raphson Method to find the root, correct to two decimal places.

35a) The dimensions of a rectangle are 8cm and 4.26.
(i) State the maximum possible error in each dimension.
(ii) Find the range within which the area of the rectangle lies. (correct to 2 decimal places)
b) The radius r and height h of a cylinder are measured with corresponding errors Δr and Δh respectively. Show that the maximum possible error in the volume is $\left|\frac{\Delta h}{\mathrm{h}}\right| + 2\left|\frac{\Delta r}{\mathrm{r}}\right|$

36a) Derive the simplest formula based on Newton Raphson's method to show that for the equation $3x = 1n3$ it satisfies $x_{r+1} = \frac{1}{3}\left\{\frac{e^{3x_r}(3x_r - 1) + 3}{e^{3x_r}}\right\}$

b) Draw a flow chart that;

i) reads the initial approximation x_0,

ii) Accepts estimations to 4 significant figures

iii) Prints the root to $4sf$.

iv) Perform a dry-run for your flow chart for $x_0 = \frac{1}{3}$.

37a) Show that the iterative formula based on Newton Raphson's Method for approximating the root of the equation $2\ln x = x - 1$ is given by;

$$x_{n+1} = x_n\left(\frac{2\ln x_n - 1}{x_n - 2}\right) \text{ n = 0, 1, 2. . . . .}$$

(b) Draw a flow chart that,

(i) Reads the initial approximation x_o of the root.

(ii) Computes and prints the root correct to two decimal places, using the formula in (a) above.

END

COMPILED BY: Theode Niyirinda, GHS 0776286483/0703033048 niyirinda@gmail.com

THE INTER SCHOOL VIRTUAL A LEVEL MATHEMATICS SEMINAR 2022.
An initiative of the Holistic elearning Platform(HeLP) Schools
Saturday 02^{nd} July 2022 (9:00 a.m) and Sunday 03^{rd} July 2022 (2:00p.m)

INSTRUCTIONS TO STUDENTS AND TEACHERS:

Dear students and teachers we would like to welcome you to participate in the forthcoming Mathematics seminar for senior six students. This is in preparation for the forthcoming final exams(UNEB) and the Mock Examinations. The seminar is organized by teachers under the Holistic eLearning programme. **This is a free seminar and no one should charge you any fees.**The process to be followed by both the teachers and students is suggested below:

1. Teachers share the Seminar questions with their students and ask for volunteers to discuss any of the questions.
2. Teachers talk to the school administrators to allow the children participate as presenters in the seminar on Saturday **02nd July from 09:00am - 2:00 pm.** Other students will just be participants.
3. There will be a repeat of this seminar on 03rd July 2022 starting at $2:00-5:00$pm to cater for the SDA community and others.
4. If your student is going to present then as the teacher(s) prepare her/him by looking through the calculations made by the student. Then encourage the student to write out the solution neatly in black pen including any graph. Then they scan or take a picture and send to the teacher . They can also type out the solution in a word or PowerPoint document and share with the teacher. The teacher or student will hand in the solutions to Kaziba Stephen (0787698238(Whats-App)) by monday **27th June 2022.** or upload the solution to the padlet https://padlet.com/holisticelearnplatform/Alevelmathematicsseminar
5. The teacher could now train the student on how to present in zoom as far as sharing a screen and using the whiteboard. Alternatively the students' presentation will be loaded on the computer screen and they explain to us their solution.

SEMINAR DETAILS

Holistic eLearning Platform is inviting you to a scheduled Zoom meeting.
Time: JULY 02, 2022 09:00 AM

This session will repeat on 03rd July 2022 at 2:00pm
Join Zoom Meeting
https://us02web.zoom.us/j/9133022418?pwd=cDlYOGNNV1hpdDU3SXBZUzdWMXorQT09
Meeting ID:9133022418
Passcode: HeLP
Kindly register your school using the survey link https://bit.ly/3sF2p7B .

P425/1	P425/2
1. Analysis (6 questions) (a) Differentiation (b) Intergration (c) Differential equations 2. Vectors (2 questions) (a) Vectors in 2-D (b) Vectors in 3-D (c) Ratio theorem (d) Line and their properties (e) Planes and their properties 3. Trigonometry (2 questions) 4. Geometry (2 questions) (a) Coordinate geometry of lines and triangles (b) Locus and circles (c) Parabola 5. Algebra (4 questions) (a) Surds,indices and logarithms (b) Quadratics (c) Polynomials (d) Simultaneous equations (e) Inequalities (f) Partial fractions (g) Complex numbers (h) Permutation and combinations	1. Mechanics (6 questions) (a) Calculus of vectors (b) General motion of the body (c) Relative motion (d) Projectiles (e) Newtonian mechanics (f) e.t.c 2. Numerical analysis (4 questions) (a) Location of the roots of an equation (b) Trapezium rule of numerical intergration (c) Newton raphson method (d) Errors (e) Flow charts 3. Statitics and probability(6 questions) (a) Mean ,node,median (b) Index numbers (c) Correlation coefficient (d) Scatter diagram (e) Discrete probability distributions (f) Continous probability distributions (g) Distributions i. Uniform distribution ii. Normal distribution iii. Binomial distribution iv. Normal approximation to binomial distribution (h) Estimations

"Yesterday's failures are today's seeds that must be diligently planted to be able to abundantly harvest tomorrow's success."

PURE MATHEMATICS (P425/1)

ALGEBRA

1. Solve the inequality $|2x+3| > 3|x+2|$

2. The polynomial $p(x)$ is defined by $p(x) = mx^3 + nx^2 - 17x - 6$, where m and n are constants. It is given that $(x+2)$ is a factor of $p(x)$ and that the remainder is 28 when $p(x)$ is divided by $(x-2)$.

   (a) Find the values of m and n .

   (b) Hence factorise $p(x)$ completely.

3. (a) Expand $\left(1+\frac{1}{4}x\right)^4$ up to the third term and use it to estimate $(1.025)^4$ correct to 3 decimal places.

   (b) Expand $(1+x)^{\frac{1}{3}}$ up to the term in x^2 . By using the substitution $x = \frac{1}{8}$, estimate the cube root of 9 correct to 3 decimal places.

   (c) Solve the equation ${}^nP_2 = 20$

4. (a) The coefficients of x^2 and x^3 in the expansion of $(3-2x)^6$ are a and b respectively. Find the value of $\frac{a}{b}$

   (b) i. Find the coefficient of x in the expansion of $\left(2x - \frac{1}{x}\right)^5$

   ii. Hence find the coefficient of x in the expansion of $(1+3x^2)\left(2x - \frac{1}{x}\right)^5$

5. (a) The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20 000.

   (b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression.

6. (a) Show that $1+i$ is a root of the equation $z^4 + 3z^2 - 6z + 10 = 0$.Hence find other roots

   (b) Given that the complex number z and its conjugate $\bar{z}$ satisfy the equation $z\bar{z} - 2z + 2\bar{z} = 5 - 4i$.Find the possible values of z

7. (a) Solve the simultaneous equations

$$m^2 - 4mn + n^2 = 1$$
$$m^2 + n^2 - \frac{17m}{4} = 0$$

   (b) Find the range of values of x for which $\frac{2x+1}{x+2} > \frac{1}{2}$

8. Solve the simultaneous equations :

   (a)

$$\log(m+n) = 0$$
$$2\log m = \log(5+n)$$

(b)

$$\begin{aligned} 2a - b + 4c &= 26 \\ 3a - 2c - b &= 0 \\ a + 3b - c &= 5 \end{aligned}$$

9. Express the following into partial fractions

(a) $\frac{3x^3-x^2+2}{x(x^2-1)}$

(b) $\frac{5x-12}{(x+2)(x^2-2x+3)}$

(c) $\frac{2x^2+x-2}{x^3(x-1)}$

(d) $\frac{x^3+5x^2+4x+5}{(x-1)(x^3-1)}$

VECTORS

10. (a) Determine the equation of the plane through the points A(1,1,2),B$(2,-1,3)$ and C$(-1,2,-2)$.

(b) A line through the point D$(-13,1,2)$ and parallel to the vector $12i+6j+3k$ meets the plane in (a) at point E.Find :

i. the coordinates of E

ii. the angle between the line and the plane

11. (a) Find the cartesian equation of the line of intersection of the two planes $2x-3y-z=1$ and $4y+3x+2z=3$

(b) Vectors **a,b** and **c** form the three sides of a triangle.Given that $|a|=5$,$|b|=12$ and **a.b**$=30\sqrt{3}$,find the area of the triangle

12. (a) The points A,B and C have position vectors **a**$=5i+3j+k$,**b**$=2i-j+3k$ and **c**$=7i-3j+10k$ respectively.Show that ABC is a triangle and find the area of triangle ABC

(b) i. Find the coordinates of the point of intersection of the line $r=2i-k+\alpha(i+3j)$ and the plane $5x-y-7z=9$

ii. Calculate the angle between a line and a plane in b(i) above

13. Two planes have equations $2x-y-3z=7$ and $x+2y+2z=0$

(a) Find the acute angle between the planes

(b) Find a vector equation for their line of intersection

TRIGONOMETRY

14. (a) Prove the identity

$$\left(\frac{1}{\cos\theta}-\tan\theta\right)\left(\sin^2+\frac{1}{\sin\theta}+\cos^2\theta\right)=\frac{1}{\tan\theta}$$

(b) Hence solve the equation $\left(\frac{1}{\cos\theta}-\tan\theta\right)\left(\sin^2+\frac{1}{\sin\theta}+\cos^2\theta\right)=2\tan^2\theta$ for $0^0\leq\theta\leq 180^0$

15. (a) Prove that $\frac{\sin\beta}{1+\cos\beta} = \tan\frac{\beta}{2}$

    (b) Solve for θ in the equation $\cos 5\theta - \cos\theta = \sin 3\theta$ in the range $0^0 \leq \theta \leq 360^0$

16. If A,B ,C are angles of a triangle ,prove that

    (a) $\sin\frac{B}{2} = \cos\left(\frac{A+C}{2}\right)$

    (b) $\cos A + \cos B + \cos C - 1 = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

    (c) $\frac{\cos A+\cos B}{\sin A+\sin B} = \tan\frac{C}{2}$

    (d) $\cos 2A + \cos 2B + \cos 2C = 1 - \cos A\cos B\cos C$

17. Prove the following identities

    (a) $\frac{1-\cos 2B}{\sin 2B} = \tan B$

    (b) $\sec 2\theta + \tan 2\theta = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

    (c) $\frac{\sin 2A}{1+\cos 2A} = \tan A$

    (d) $\cos^4\beta - \sin^4\beta = \cos 2\beta$

    (e) $(\cos x + \mathrm{cosec} x)^2 = \frac{1+\cos x}{1-\cos x}$,where $\cos x \neq -1$

18. (a) Solve $2\sin 2\theta = 3\cos\theta$ for $-180^0 \leq \theta \leq 180^0$

    (b) Solve $\sin\theta - \sin 4\theta = \sin 2\theta - \sin 3\theta$ for $-\pi \leq \theta \leq \theta$

19. (a) Solve $\cos\theta + \sqrt{3}\sin\theta = 2$ for $0^0 \leq \theta \leq 360^0$

    (b) Solve for β if $\cos\beta + \cos 5\beta + \cos 3\beta = 0$ for $0^0 \leq \beta \leq \pi$

    (c) If P,Q and R are angles of a triangle prove that :

$$\frac{1}{p}\cos^2\left(\frac{P}{2}\right) + \frac{1}{q}\cos^2\left(\frac{Q}{2}\right) + \frac{1}{r}\cos^2\left(\frac{R}{2}\right) = \frac{(p+q+r)^2}{4pqr}$$

GEOMETRY

20. The equation of a circle is $x^2 + y^2 + ax + by - 12 = 0$.The points A(1,1) and B$(2,-6)$ lie on the circle

    (a) Find the values of a and b and hence find the coordinates of the centre of the circle

    (b) Find the equation of the tangent to the circle at the point A,giving your answer in the form $px + qy = k$,where p,q and k are integers

21. The equation of a circle is $x^2 + y^2 - 8x + 4y + 4 = 0$.

    (a) Find the radius of the circle and the coordinates of its centre

    (b) Find the x- coordinates of the points where the circle crosses the x-axis giving your answers in exact form

    (c) Show that the point B$(6, 2\sqrt{3} - 2)$ lies on the circle

    (d) Show that the equation of the tangent to the circle at B is $\sqrt{3}x + 3y = 12\sqrt{3} - 6$

22. (a) $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ are points on the parabola $y^2 = 4ax$.If the chord passes through the focus ,show that $pq = -1$.If M is the midpoint of PQ ,deduce that the locus of M is $y = 2a(x - a)$

(b) Show that the equation $y^2 = 9(x + y)$ represents a parabola;hence determine its focus and directrix

23. (a) A tangent from the point $Q(q^2, 2q)$ touches the curve $y^2 = 4x$ .Find

i. the equation of the tangent

ii. the equation of line L parallel to the normal at $(q^2, 2q)$ and passes through $(1, 0)$

iii. The point of intersection , P of the line L and the tangent

(b) A point R(x,y) is equidistant from P and Q in (a) above .Show that the locus of R is $q^4 - 3q = 2(x + y)$

ANALYSIS

24. A curve has equation $y = 7 + 4In(2x + 5)$ Find the equation of the tangent to the curve at the point $(-2, 7)$, giving your answer in the form $y = mx + c$.

25. (a) Given that $y = \tan 2x$, show that $\frac{dy}{dx} = 2\tan x + 2\tan^3 x$

(b) Show that the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\tan x + \tan^2 x + \tan^3 x)dx = \sqrt{3} - \frac{\pi}{2}$

26. The parametric equations of a curve are $x = 1 - \cos\beta$,$y = \cos\beta - \frac{1}{4}\cos 2\beta$.Show that

$$\frac{dy}{dx} = -2\sin^2\left(\frac{1}{2}\beta\right)$$

27. (a) Differentiate $\frac{x^3}{\sqrt{(1-2x^2)}}$ with respect to x

(b) The period ,T of a swing of a simple pendulum of length ,l is given by the equation $T^2 = \frac{4\pi^2 l}{g}$,where g is the acceleration due to gravity.An error of 2% is made in measuring the length ,l .Determine the resulting percentages error in the period ,T

(c) If $y = 3x^2 - x$ .show that $y\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y + 1 = 6x$

28. (a) A curve is given by the equation $y = x^2 + 1$. Find the area bounded by the curve, the axes and the line$x = 2$.

(b) Find the volume of the solid generated when this area is revolved about the ;

i. x − axis.

ii. y − axis.

29. (a) Find $\int(2\cos\theta - 3)(\cos\theta + 1)d\theta$

(b) Prove the identity

$$\frac{1 - \tan^2\theta}{1 + \tan^2\theta} = 2\cos^2\theta - 1$$

30. (a) Prove each of these identities

i. $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{2}{\sin\theta}$

ii. $\frac{\cos\theta}{\tan\theta(1+\sin\theta)}=\frac{1}{\sin\theta}-1$

(b) Hence solve the equation $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=1+3\sin\theta$ for $0^0\leq\theta\leq 360^0$

31. (a) If $y=In(x^2-5)$,show that $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=2e^{-y}$

(b) If $y=e^x\sin x$,show that $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

32. (a)

$$\int xIn(x^2-25)dx$$

(b) Evaluate

$$\int_0^2\frac{dx}{x^2\sqrt{16-x^2}}$$

33. (a) Differentiate the following from first principles

i. $\sin^2 2x$

ii. $x10^{\sin x}$

(b) Given that $y=In\left(e^{2x}\left(\frac{x+3}{x-3}\right)\right)^{\frac{-2}{3}}$.Find $\frac{dy}{dx}$

34. (a) Solve the equations below

i.

$$(1+x^2)\frac{dy}{dx}-y(y+1)x=0,\text{given that y=1 when x=0}$$

ii.

$$\frac{dy}{dx}-y\tan x-\cos x=0,\text{given that y=0 at } x=\frac{\pi}{2}$$

(b) The rate at which the temperature of a body falls is proportional to the difference between the temperature of the body and that of its surrounding .Initially the temperature of the body is 80^0C .After 10 minutes the temperature of the body is 60^0C .The temperature of the surrounding is 15^0C

i. Form a differential equation for the temperature of the body

ii. Determine the time it takes for the temperature of the body to reach 40^0C

35. Maize dwarf mosaic virus(MDMV) has infected a number of maize plants Mr Ronalds' garden .The growth in the number of maize plants infected is proportional to the number already infected .Initially 20 maize plants were infected

(a) Form a differential equation that models the growth in the number infected

(b) Thirty days after the initial number of infections ,60 maize plants were infected .After how many further days does the model predict that 200 maize plants will be infected?

36. Given that

(a) $y = \sin(log_e x)$,show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(b) $y = e^{2x}\cos 3x$,show that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$

37. (a) Sketch on the same diagram,the curves $y = x^2 - 5x$ and $y = 3 - x^2$,and find their points of intersection

    (b) Find the area of the region bounded by the two curves

38. (a) Find the quotient and remainder when $8x^3 + 4x^2 + 2x + 7$ is divided by $4x^2 + 1$.

    (b) Hence find the exact value of $\int_0^{\frac{1}{2}} \frac{8x^3+4x^2+2x+7}{4x^2+1}dx$

39. (a) Find $\int x^3 e^{x^4} dx$

    (b) Use the substitution $t = \tan\theta$ to find $\int \frac{1}{1+\sin^2\theta}d\theta$

    (c) Find the area enclosed by the line $y = x + 1$ and the curve $y = x^2 - 2x - 3$

40. (a) Express $\frac{8-6x}{(x+3)(x^2+4)}$ in partial fractions

    (b) Hence show that $\int_1^2 \frac{8-6x}{(x+3)(x^2+4)}dx = In\left(\frac{125}{128}\right)$

41. At time t days ,the rate of decay of the mass of a radioactie material is proportional to the mass,m grams ,of the radioactive material that is present at that time .At $t = 0$,$m = 150g$ and $t = 5, m = 100g$.

    (a) Show that $m = 150e^{-\frac{1}{5}In(\frac{3}{2})t}$

    (b) Find the length of time that it takes for the mass of the radioactive material present to be halved

42. Intergrate with respect to x the functions.

    (a) $\int \frac{6x-10}{(2x+1)^2}dx$

    (b) $\int \frac{6x-2x^2-8}{(x-1)(x^4-1)}dx$

    (c) $\int \sin^{-1} x$d

    (d) $\int \frac{\cos\theta}{\cos 2\theta}d\theta$

APPLIED MATHEMATICS (P425/2)

MECHANICS

43. A particle M is projected vertically upwards from horizontal ground with speed u ms $^{-1}$. M reaches a maximum height of 20m above the ground.

    (a) Find the value of u .

    (b) Find the total time for which M is at least 15 m above the ground.

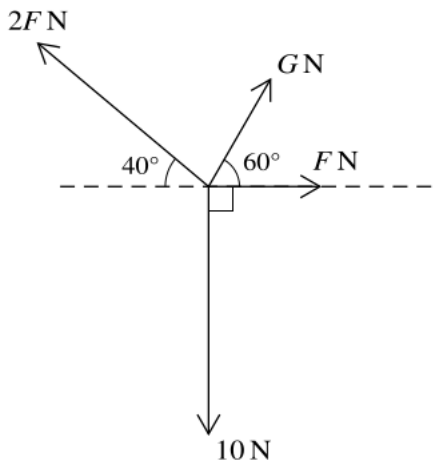
44. A cyclist starts from rest at a fixed point O and moves in a straight line, before coming to rest k seconds later. The acceleration of the cyclist at time t s after leaving O is a ms^{-2} , where a = $\frac{2}{\sqrt{t}} - \frac{3\sqrt{t}}{5}$ for $0 < t \leq k$

    (a) Find the value of k .

    (b) Find the maximum speed of the cyclist.

(c) Find an expression for the displacement from O in terms of t . Hence find the total distance travelled by the cyclist from the time at which she reaches her maximum speed until she comes to rest.

45. Four coplanar forces act at a point. The magnitudes of the forces are 10 N, F N, G N and 2 F N. The directions of the forces are as shown in the diagram.



(a) Given that the forces are in equilibrium, find the values of F and G .

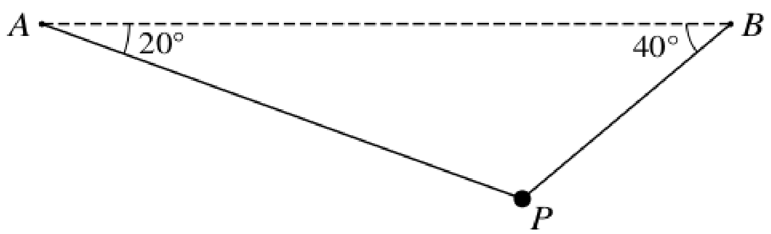
(b) Given instead that F = 3, find the value of G for which the resultant of the forces is perpendicular to the 10 N force.

46. A particle A is projected with speed 20 m s^{-1} at an angle of 60^0 below the horizontal, from a point O which is 30 m above horizontal ground.

(a) Calculate the time taken by A to reach the ground.

(b) Calculate the speed and direction of motion of A immediately before it reaches the ground.

47. A particle P of mass 1.6 kg is suspended in equilibrium by two light inextensible strings attached to points A and B . The strings make angles of 20^0 and 40^0 respectively with the horizontal . Find the tensions in the two strings.



48. A particle P moves in a straight line, starting from rest at a point O on the line.At time t s after leaving O the acceleration of P is $k(16-t^2)$ ms^{-2} , where k is a positive constant, and the displacement from O is s m. The velocity of P is 8 ms^{-1} when t = 4.

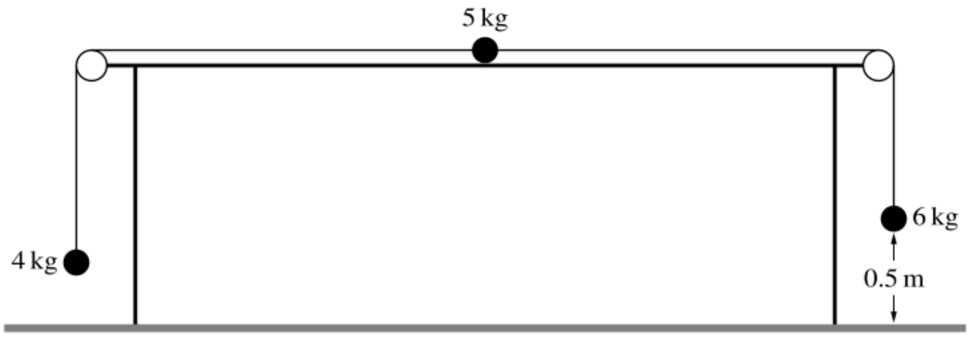
(a) Show that

$$s = \frac{1}{64}t^2(96 - t^2)$$

(b) Find the speed of P at the instant that it returns to O .

(c) Find the maximum displacement of the particle from O .

49. The diagram below shows a particle of mass 5 kg on a rough horizontal table, and two light inextensible strings attached to it passing over smooth pulleys fixed at the edges of the table. Particles of masses 4 kg and 6 kg hang freely at the ends of the strings. The particle of mass 6 kg is 0.5 m above the ground. The system is in limiting equilibrium.



(a) Find the coefficient of friction between the 5 kg particle and the table

(b) The 6 kg particle is now replaced by a particle of mass 8 kg and the system is released from rest. Find the acceleration of the 4 kg particle and the tensions in the strings.

50. Two uniform rods AB and BC of the same length and masses 8kg and 6kg respectively are smoothly joined at B.The rods A and C rest on a rough horizontal plane.The rods are inclined at 120^0 to each other when BC is on the point of slipping.Find the:

(a) Normal reactions at A and C

(b) Coefficient of friction between the ground and rod BC

(c) Magnitude of the reaction at B

51. At time $t = 0$,the position vector **r** and velocity **v** of two trains A and B are as follows.

Trains	Velocity vector	Position vector
A	$V_A = (-6i + k)ms^{-1}$	$r_A = (i + 2j + 3k)m$
B	$V_B = (-5i + j + 7k)ms^{-1}$	$r_B = (4i - 14j + k)m$

If the trains maintain these velocities,find the :

(a) Position of B relative to A at time t

(b) time that elapses before the trains are closest to each other

(c) least distance between the trains in the subsequent motion

52. A particle of mass $3kg$ is acted upon by a force $F = 6i - 3t^2j + 27tk$ newtons at time t.At time $t = 0$,the particle is at the point with a position vector $i - 5j - k$ and its velocity is $(3i + 3j)\text{ms}^{-1}$.Determine the:

(a) Position vector of the particle at time $t = 1$second.

(b) distance of the particle from the origin at time $t = 2$seconds

NUMERICAL ANALYSIS

53. Given that f(0.9) = 0.2661, f(1.0) = 0.2420 and f(1.1.) = 0.2179, estimate:
    (i) f(0.94)
    (ii) $f^{-1}(0.2362)$ using linear interpolation or extrapolation

54. (a) Given that the equation $e^x = 10 - x$ has a root between 2 and 3.Show that the iterative formula based on Newton Raphson method is given as
    $$x_{(n+1)} = \frac{10 + e_n^x(x_n - 1)}{e^{x_n} + 1}$$
    (b) Draw a flowchart that reads,prints the root x and the number of iterations.Carry a dry run of the flowchart and obtain a root with an error of less than 0.001,taking the initial approximation $x_0 = 2.45$

55. (a) i. On the same axes ,draw graphs of $y = e^{2x}$ and $y = 5x + 1$ for $-0.5 \leq x \leq 1$
    ii. From your graphs ,obtain to 1 significant figure ,an appropriate root of the equation $e^{2x} - 5x - 1 = 0$.
    (b) Using the Newton -Raphson method ,find the root of the equation $e^{2x} - 5x - 1 = 0$ taking the appropriate root in (a) as an initial approximation .Give your answer correc to 3 decimal places.

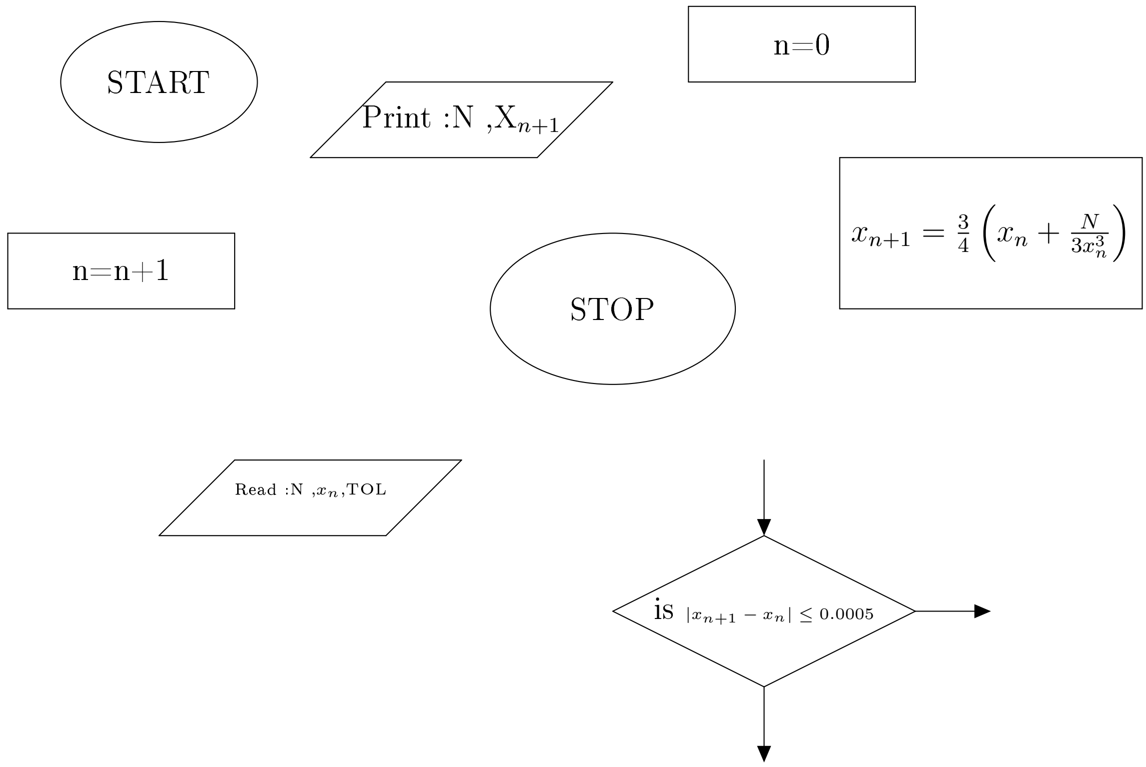
56. (a) Use the trapezium rule with 7 ordinates to estimate $\int_0^6 xe^{-x}dx$ correct to 3 significant figures
    (b) Find the percentage error made in your estimation ,giving your answer to 2 deciml places .Suggest how this error may be reduced.

57. Given the numbers $p = 30.75$ and $q = 6.125$ all measured to their nearest number of decimal places indicated
    (a) State the maximum possible errors in p and q
    (b) Find the absolute error in the quotient $\frac{p}{q}$
    (c) Find the limits within which the exact value of the quotient $\frac{p}{q}$ lies

58. (a) The radius r and height h of a cylinder were measured with errors δr and δh respectively.Show that the maximum relative error in the volume of the cylinder is $2|\frac{\delta r}{r}| + |\frac{\delta h}{h}|$
    (b) The quantities x and y were measured with errors δx and δy respectively.Show that the maximum relative error in $\sqrt{xy}$ is $\frac{|y\delta x| + |x\delta y|}{2|xy|}$ hence find the limits within which $\sqrt{2.4 \times 4.86}$ is expected to lie

59. (a) Use the trapezium rule with three strips to estimate the integral $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1+\cos x}}$,correct to three decimal places
    (b) Calculate the percentage error in your estimation in (a) above

60. Given below are parts of a flow chart not arranged in order



(a) Re arrange them and draw a complete logical flow chart

(b) Using N=44 and $x_0 = 2$Perform a dry run of your arranged flow chart

(c) State the purpose of the flow chart

61. The table below shows the values of;

θ	$0'$	$6'$	$12'$	$18'$	$24'$	$30'$
$\sin 10^0$	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822

Use linear interpolation to find,

(a) $\sin 10^0 16'$

(b) $\sin^{-1} 0.1747$

STATISTICS AND PROBABILITY

62. (a) Stephen keeps layer birds.Its assumed that the birds are likely to lay a yellow or white yolked egg.Find the probability that in the 5 eggs picked there will be more yellow yolked eggs than white yolked eggs

(b) Calculate an estimate of the mean height of 50 tomato plants , where there are sixteen 18cm height, twenty 19cm height and fourteen that are either 20 or 21 cm height.

63. Pack A consists of ten cards numbered 0, 0, 1, 1, 1, 1, 1, 3, 3, 3. Pack B consists of six cards numbered 0, 0, 2, 2, 2, 2. One card is chosen at random from each pack. The random variable X is defined as the sum of the two numbers on the cards.

(a) Show that $P(X = 2) = \frac{2}{15}$

(b) Draw up the probability distribution table for X .

(c) Given that $X = 3$, find the probability that the card chosen from pack A is a 1.

64. A random variable X has probability density function given by

$$f(x) = \begin{cases} 6x(1-x) & ; 0 \leq x \leq 1 \\ 0 & ; \text{otherwise} \end{cases}$$

(a) Find the probability that X does not lie between 0.3 and 0.7.

(b) Sketch the graph of the probability density function and hence state the value of $E(X)$

(c) Find Var(X)

65. A security code consists of 2 letters followed by a 4-digit number. The letters are chosen from $\{A, B, C, D, E\}$ and the digits are chosen from $\{1, 2, 3, 4, 5, 6, 7\}$. No letter or digit may appear more than once. An example of a code is BE3216.

(a) How many different codes can be formed?

(b) Find the number of different codes that include the letter A or the digit 5 or both.

(c) A security code is formed at random. Find the probability that the code is DE followed by a number between 4500 and 5000.

66. The times taken, in minutes, by 360 employees at Mehta Sugar Factory in Lugazi to travel from home to work are summarised in the following table.

Time, t minutes	$0 \leq t < 5$	$5 \leq t < 10$	$10 \leq t < 20$	$20 \leq t < 30$	$0 \leq t < 50$
Frequency	23	102	135	76	24

(a) Calculate an estimate of the mean time taken by an employee to travel to work.

(b) Draw a histogram to represent this information.

67. The cost of making a well formulated feed for the layer birds on Mr Ronald's Poultry farm is calculated from the cost of Maize bran,Broken maize ,lime and concentrate .The table below gives the cost of these items in 2021 and 2022.

ITEMS	Price(UGX) in 2021	Price (UGX) in 2022	Weight
Maize bran/kg	500	735	12
Lime/kg	500	600	2
Broken maize/kg	800	1400	5
Concentrate/kg	196000	215600	1

Using 2021 as the base year

(a) Calculate the price relative for each item hence find the simple price index for the cost of making a complete feed

(b) Find the weighted aggregate price index for the cost of the feed

68. A bag contains 5 red and 4 blue marbles. Three marbles are selected at random from the bag,without replacement.

    (a) Show that the probability that exactly one of the marbles is red is $\frac{5}{14}$

    (b) The random variable X is the number of red marbles selected.Draw up the probability distribution table for X .

    (c) Find E(X)

69. The times taken, in minutes, by farmers to plant maize on a piece of land is normally distributed with mean 32.2 and standard deviation 9.6.

    (a) Find the probability that a randomly chosen farmers takes more than 28.6 minutes to complete the task.

    (b) 20% of the farmers take longer than t minutes to complete the task. Find the value of t .

    (c) Find the probability that the time taken to complete the task by a randomly chosen farmer differs from the mean by less than 15.0 minutes.

70. (a) M and N are two independent events such that $P(M) = \frac{2}{10}$ and $P(N) = \frac{3}{20}$,Find

        i. $P(M/N)$

        ii. Probability of M or N but not both

    (b) Two events M and N are such that $P(M) = \frac{7}{10}$,$P(M \cap N) = \frac{9}{20}$ and $P(\overline{M} \cap \overline{N}) = \frac{9}{50}$ .Find

        i. $P(\overline{N})$

        ii. P(M or N but not both M and N)

71. A continuous r.v X is uniformly distributed in the interval $[a, b]$.The lower quartile is 5 and the upper quartile is 9 .Find the

    (a) Values of **a** and **b** ,hence the probability density function

    (b) E(X) and Var(X)

    (c) $P(2 < X < 10)$

72. (a) The probability that a student passes Algebra is $\frac{3}{4}$ and the probability that she passes Vectors is $\frac{13}{20}$,the probability that she passes neither of the sections is $\frac{1}{5}$ .Find the probability that she passes Algebra but not Vectors.

    (b) A bag contains 4 red balls,3 white balls and 1 yellow ball.Two balls are picked in succession at random without replacement .Find the probability that :

        i. Both are of the same color

        ii. Atleast one black ball is picked

73. An experiment consists of removing 2 sweets one at a time without replacemnt from a box containing 3 red and 4 blue sweets.

(a) If M is the event that both sweets picked are of the same color ,find the probability that event M occured

(b) If the experiment is repeated 40 times ,find the probability that event M occured

i. between 20 and 35 times

ii. atleast 24 times

74. The continuous random variable X has probability density function $f(x)$ defined by

$$f(x) = \begin{cases} \frac{m}{x^4} & x < -1 \\ m(2 - x^2) & -1 \leq x \leq 1 \\ \frac{m}{x^4} & x > 1 \end{cases}$$

(a) Find the value of constant m

(b) Determine the expected value of X and its standard deviation

75. During the National Agriculture Education show ,two Agriculture experts Mr Brian and Mr Mathias were choosen to judge the tomatoes presented by 8 schools in a prize competition for the best school in Agriculture in Uganda.

Items	A	B	C	D	E	F	G	H
Mr Mathias	70	59	71	38	55	62	80	76
Mr Brian	67	42	85	51	39	97	81	70

Calculate the rank correlation coefficient between the scores and comment on your result at 5% level of significance

76. The continuous random variable X has probability distribution defined by

$$F(x) = \begin{cases} 0 & 0 \leq x \\ \frac{x^2}{4} & 0 \leq x \leq 1 \\ mx - \frac{1}{4} & 1 \leq x \leq 2 \\ n(5 - x)(x - 1) & 2 \leq x \leq 3 \\ 1 & x \geq 3 \end{cases}$$

(a) Find the values of m and n

(b) Determine the

i. p.d.f ,f(x) of x

ii. $P(1 \leq 2x \leq 4)$

iii. E(x)

77. The table below shows the scores obtained by a random sample of 80 learners who st for the mathematics contest

(a) Calculate the :

i. Mean

Scores	Number of learners
$20- < 25$	20
$25- < 30$	12
$30- < 35$	13
$35- < 45$	4
$45- < 55$	16
$55- < 75$	15

ii. Median

(b) Display the data on a histogram and use it to estimate the mode

78. The table below shows the distribution of weights of a certain type of fruit

Weight(grams)	Frequency
20−	4
30−	3
40−	21
50−	50
60−	14
70−	8

(a) Calculate the :

i. Mean weight

ii. Standard deviation

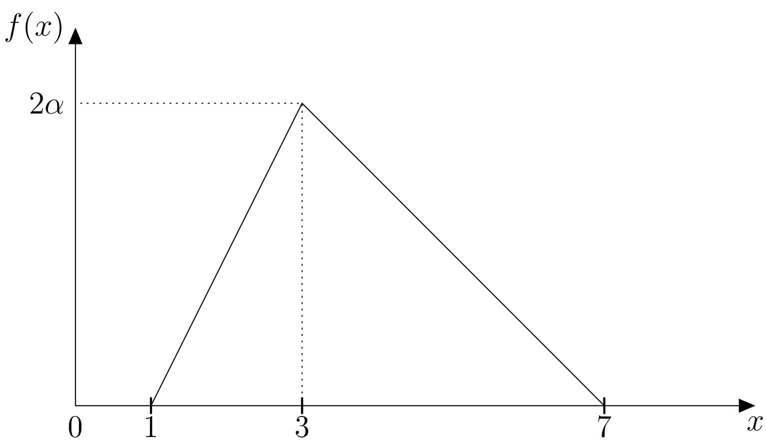
(b) Plot a cumulative frequency curve for the data and use it to estimate the

i. Median

ii. 80^{th} percentile

iii. quartile deviation

79. The p.d.f f(x) of a r.v X takes on the form shown in the sketch below



Find the

(a) Value of α

(b) Equations of the p.d.f hence the p.d.f

(c) Cumulative distribution F(x) and sketch it

(d) Median of X

80. Two locally developed antiviral drugs to cure COVID-19 were used to treat 12 symptomatic people in rotation of days and the results for the effectiveness of the herbal drugs was recorded in percentage as follows.

Covylice-1(x)	58	52	48	30	48	20	32	50	38	12	36	12
Covidex(y)	90	72	60	38	70	35	33	64	48	24	50	18

(a) Plot a scatter diagram for the data .Comment on the relationship between the two drugs

(b) Draw a line of best fit for the scatter diagram ,hence find x when $y = 68$

(c) Calculate the rank correlation coefficient for the effectiveness of the drugs .Comment on your result at 1% level of significance

END

1. Algebra

(a) For the quadratic equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(b) For an arithmetic series (A.P)

$$u_n = a + (n-1)d$$
$$S_n = \frac{1}{2}n\{2a + (n-1)d\}$$

(c) For a geometric series (G.P)

$$u_n = ar^{n-1}$$
$$S_n = \frac{a(1-r^n)}{1-r} \qquad r \neq 1$$
$$S_\infty = \frac{a}{1-r} \qquad |r| < 1$$

(d) Binomial expansion

$$(a+b)^n = a^n + \begin{pmatrix} n \\ 1 \end{pmatrix} a^{n-1}b + \begin{pmatrix} n \\ 2 \end{pmatrix} a^{n-2}b^2 + \begin{pmatrix} n \\ 3 \end{pmatrix} a^{n-3}b^3 + \cdots + b^n$$

where n is a positive integer

$$\begin{pmatrix} n \\ r \end{pmatrix} = \frac{n!}{r!(n-r)!}$$
$${}^nC_r = \frac{n!}{r!(n-r)!}$$
$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1(n-2)}{3!}x^3 \cdots$$

Where n is rational and $|x| < 1$

$${}^nP_r = \frac{n!}{(n-r)!}$$

Where $r \leq n$

(e) Summations

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$
$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$
$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

2. Trigonometry

No	**Identity**
1	$\tan\theta = \frac{\sin\theta}{\cos\theta}$,$\sec\theta = \frac{1}{\cos\theta}$,$co\sec\theta = \frac{1}{\sin\theta}$
2	$\cos^2\theta + \sin^2\theta = 1$
3	$1 + \tan^2\theta = \sec^2\theta$
4	$\cot^2\theta + 1 = co\sec^2\theta$
5	$\sin(A+B) = \sin A\cos B + \cos A\sin B$
6	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
7	$\cos(A+B) = \cos A\cos B - \sin A\sin B$
8	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
9	$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$
10	$\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$
11	$\sin 2A = 2\sin A\cos A$
12	$\cos 2A = \cos^2 A - \sin^2 A$
13	$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

(a) For the t -formula

$$t = \tan\frac{1}{2}\theta$$
$$\sin\theta = \frac{2t}{1+t^2}$$
$$\cos\theta = \frac{1-t^2}{1+t^2}$$

(b) For any triangle with angles ,A,B and C and with sides a,b,and c .

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \text{Cosine rule}$$
$$s = \frac{a+b+c}{2}$$

3. Differentiation

No	y	$\frac{dy}{dx}$
1	x^n	nx^{n-1}
2	$\ln x$	$\frac{1}{x}$ for $x \neq 0$
3	e^x	e^x
4	$\sin x$	$\cos x$
5	$\cos x$	$-\sin x$
6	$\tan x$	$\sec^2 x$
7	uv	$u\frac{dv}{dx} + v\frac{du}{dx}$
8	$\frac{u}{v}$	$\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
9	$f(x)$	$\frac{f(x+\delta x)-f(x)}{\delta x}$
10	$\sec x$	$\sec x\tan x$
11	y=u,u=x	$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$
12	$e^{f(x}$	$f^1(x)e^{f(x)}$

4. Integration

No	$f(x)$	$\int f(x)dx$
1	x^n	$\frac{x^{n+1}}{n+1} + c$ for $n \neq -1$
2	$\frac{1}{x}$	$\ln\|x\| + c$ n
3	e^x	$e^x + c$
4	$\sin x$	$-\cos x + c$
5	$\cos x$	$\sin x + c$
6	$\sec^2 x$	$\tan x + c$
7	$\int u\frac{dv}{dx}dx$	$uv - \int v\frac{du}{dx}dx$
8	$\int \frac{f^1(x)}{f(x)}dx$	$\ln\|f(x)\| + c$
9	$co\sec x \cot x$	$-co\sec x + c$
10	$\sec x \tan x$	$\sec x + c$
11	$co\sec^2 x$	$-\cot x + c$
12	$\tan x$	$\ln\|\sec x\| + c$
13	$co\sec x$	$-\ln\|co\sec x + \cot x\| + c$
14	$\cot x$	$\ln\|\sin x\| + c$

(a)

$$\int \frac{1}{a^2 - b^2x^2}dx = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right) + c$$

(b)

$$\int \frac{1}{a^2 + b^2x^2}dx = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right) + c$$

(c)

$$\int \frac{a}{p + qx}dx = \frac{a}{q}\ln|p + qx| + c$$

5. Vectors

(a) If $a = a_1i + a_2j + a_3k$ and $b = b_1i + b_2j + b_3k$ then

$$\begin{aligned} a.b &= a_1b_1 + a_2b_2 + a_3b_3 \\ &= |a||b|\cos\theta \end{aligned}$$

(b) $i.i = j.j = k.k = 1$ and $i.j = i.k = j.k = 0$

(c) $|a.a| = |a|^2$

(d) $a.(b + c) = a.b + a.c$

(e) $a.(kb) = (ka).b = k(a.b)$ where k is a constant

(f) $a.b = |a||b|\cos\theta$

(g) The cartesian equation of the line

$$\frac{x - a}{x_1} = \frac{y - b}{y_1} = \frac{z - c}{z_1}$$

1. Numerical Methods

(a) Trapezium rule

$$\int_a^b f(x)dx \approx \frac{1}{2}h\{y_o + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n\}$$

Where $h = \dfrac{b-a}{n}$

(b) Newton Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)} \qquad \text{Where } x = 0, 1, 2, \cdots$$

(c) Ordinates and sub intervals

The number of sub intervals = Number of ordinates − 1

(d) The maximum possible error made due to rounding off is given by

$$\text{Error} = 0.5 \times 10^{-n}$$

Where **n** is the number of decimal places rounded off to

(e) Error

$$\text{Error } = \text{Exact value} - \text{Approximate value.}$$

(f) Absolute error This is the actual size of the error and is always positive .It is the magnitude of the error

$$\text{Error } = |\text{Exact value} - \text{Approximate value}|.$$

(g) Relative error

$$\text{Relative Error } = \frac{\text{Absolute error}}{\text{Exact value}}$$

The relative error must always be positive

$$\text{Relative Error } = \frac{|\text{Error}|}{\text{Exact value}}$$

(h) Percentage error

$$\text{Percentage Error } = \frac{|\text{Error }|}{\text{Exact value}} \times 100$$

(i) The interval or range with in which the exact value lies is given by Min value≤ Exact value ≤ Max value or [Min,Max]

(j) Absolute error $= \frac{\text{Maximum value} - \text{Minimum value}}{2}$

2. Probability and Statistics

(a) The mean for ungrouped data is calculated using the formula

$$\text{Mean} = \frac{\text{sum of data values}}{\text{number of values in the data}}$$
$$\overline{X} = \frac{\sum x}{n}$$

(b) The mean for grouped data is calculated using the formula

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

Where **x** is the class mark and **f** is the frequency

(c) The mean for grouped data when given an assumed means is calculated using the formula

$$\text{Mean} = A + \frac{\sum fd}{\sum f}$$

Where **A** is the assumed mean or working mean

d is the deviation given as $d = x - A$

(d) For grouped data ,the median is calculated using

$$\text{Median} \ = L_1 + \left(\frac{\frac{\sum f}{2} - CF_b}{f_m} \right) \times C$$

Where

$$\begin{aligned} L_1 &= \text{Lower class boundary of the median class} \\ CF_b &= \text{Cummulative frequency before the median class} \\ f_m &= \text{frequency within the median class} \\ C &= \text{Class width} \\ \sum f &= \text{Total frequency} \end{aligned}$$

(e) For grouped data with equal class width the mode is calculated using

$$\text{Mode} = L_1 + \left(\frac{d_1}{d_1 + d_2} \right) \times C$$

Where

$$\begin{aligned}
L_1 &= \text{Lower class boundary of the modal class} \\
d_1 &= \text{Modal frequency } - \text{Pre modal frequency} \\
d_2 &= \text{Modal frequency } - \text{Post modal frequency} \\
C &= \text{Class width}
\end{aligned}$$

(f) For grouped data ,the lower quartile is calculated using

$$q_1 = L_1 + \left(\frac{\frac{\sum f}{4} - CF_b}{f_m} \right) \times C$$

Where

$$\begin{aligned}
L_1 &= \text{Lower class boundary of the } q_1 \text{ class} \\
CF_b &= \text{Cummulative frequency before the } q_1 \text{ class} \\
f_m &= \text{frequency within the } q_1 \text{ class} \\
C &= \text{Class width} \\
\sum f &= \text{Total frequency}
\end{aligned}$$

(g) For grouped data ,the upper quartile is calculated using

$$q_3 = L_1 + \left(\frac{\frac{3\sum f}{4} - CF_b}{f_m} \right) \times C$$

Where

$$\begin{aligned}
L_1 &= \text{Lower class boundary of the } q_3 \text{ class} \\
CF_b &= \text{Cummulative frequency before the } q_3 \text{ class} \\
f_m &= \text{frequency within the } q_3 \text{ class} \\
C &= \text{Class width} \\
\sum f &= \text{Total frequency}
\end{aligned}$$

(h) Inter quartile range $= q_3 - q_1$

(i) For grouped data ,the variance is calculated using

$$\text{Var(x)} = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2$$

(j) Standard deviation $= \sqrt{\text{Var(x)}}$

3. Index numbers

(a)

$$\text{Price relative } = \frac{p_n}{p_0} \times 100$$

Where

p_n = Price of the commodity in the given year(new year)

p_0 = Price of the commodity in the base year(old year)

(b)

$$\text{Simple price index} = \frac{\text{Sum of the price relatives}}{\text{Number of items (N)}} \times 100$$
$$= \frac{\sum \left(\frac{p_n}{p_0}\right) \times 100}{N}$$

(c)

$$\text{Simple aggregate price index} = \frac{\text{Current year price total}}{\text{Base price total}} \times 100$$
$$= \frac{\sum p_n}{\sum p_0} \times 100$$

(d)

$$\text{Weighted price index} = \text{Price relatives} \times \text{weights}$$
$$= \frac{p_n}{p_o} \times w \times 100$$

(e)

$$\text{Weighted aggregate price index} = \frac{\sum p_n w}{\sum p_0 w} \times 100$$

(f)

$$\text{Weighted average price index} = \frac{\sum \frac{p_n}{p_0} \times 100 \times w}{\sum w}$$

(g)

$$\text{Value index} = \frac{\sum p_n q_n}{\sum p_o q_o} \times 100$$

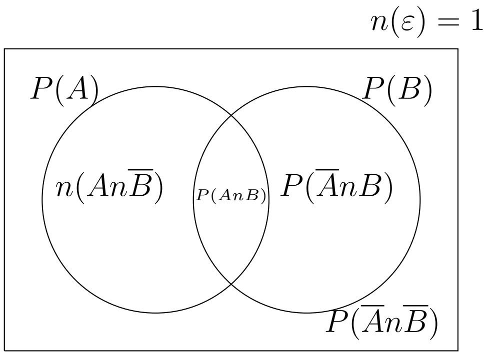
4. Spearman's rank correlation coefficient

$$\rho = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

Where n is the difference between the rankings of a given scores and n is the number of pairs

5. Probability theory

(a) For any events A and B



$$P(A) = P(A \cap \overline{B}) + P(A \cap B)$$
$$P(\overline{A}) = P(\overline{A} \cap B) + P(A \cup B)^1$$
$$P(B) = P(\overline{A} \cap B) + P(A \cap B)$$
$$P(\overline{B}) = P(A \cap \overline{B}) + P(A \cup B)^1$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(b) $P(A) + P(\overline{A}) = 1$

(c) $P(A \cup B)^1 = P(\overline{A} \cap \overline{B})$

(d) $P(\overline{A} \cup \overline{B}) = P(A \cap B)^1$

(e) $P(A/B) = \frac{P(A \cap B)}{P(B)}$ for $P(B) \neq 0$

6. Mechanics

(a) For projectile motion

$$y = x\tan\theta - \frac{gx^2}{2u^2}sec^2\theta$$

(b) For calculus

Physical quantity	Formula	units	Formula	units	Formula
Force	F=ma	N	$a = \frac{dv}{dt}$	ms^{-2}	k.e=$\frac{1}{2}mv^2$
Power	P=F.v	W	$v = \frac{dr}{dt} or \frac{ds}{dt}$	ms^{-1}	Avg accel=$\frac{v(t_2)-v(t_1)}{t_2-t_1}$
Work done	W=F.s or F.r	j	W=$\int_{t_1}^{t_2} f.vdt$	j	speed=$\|v\|$
Impulse	I=F.t	Ns	$v = \int adt$	ms^{-1}	Avg vel=$\frac{r(t_2)-r(t_1)}{t_2-t_1}$
Momentum	momentum =m.v	$Kgms^{-1}$	$r = \int vdt$	m	distance=$\|r\|or\|s\|$

SIGNIFICANCE LEVELS FOR CORRELATION COEFFICEINTS

	Product-moment coefficient of correlation (r_{xy})		Spearman's rank Correlation coefficient (ρ)		Kendall's rank coefficient of correlation (τ)	
No. of pairs	Significance if $\lvert r_{xy} \rvert$ exceeds at 5%	at 1%	Significance if $\lvert \rho \rvert$ exceeds at 5%	at 1%	Significance if $\lvert \tau \rvert$ exceeds at 5%	at 1%
3	1.00	1.00				
4	0.95	0.99				
5	0.88	0.96	1.00			
6	0.81	0.92	0.89	1.00	0.87	1.00
7	0.75	0.88	0.75	0.89	0.71	0.81
8	0.71	0.83	0.71	0.86	0.64	0.79
9	0.67	0.80	0.68	0.83	0.56	0.72
10	0.63	0.77	0.65	0.79	0.51	0.64
11	0.60	0.74	0.60	0.74	0.49	0.60
12	0.58	0.71	0.58	0.71	0.45	0.58
13	0.55	0.68	0.55	0.68		
14	0.53	0.66	0.53	0.66		
15	0.51	0.64	0.51	0.64		
16	0.50	0.62	0.50	0.62		
17	0.48	0.61	0.48	0.61		
18	0.47	0.59	0.47	0.59		
19	0.46	0.58	0.46	0.58		
20	0.44	0.56	0.44	0.56	0.33	
30	0.35	0.45	0.35	0.45		
40	0.31	0.39	0.31	0.39		
50	0.27	0.35	0.27	0.35		
60	0.25	0.33	0.25	0.33		
70	0.23	0.31	0.23	0.31		
80	0.22	0.29	0.22	0.29		
90	0.21	0.27	0.21	0.27		
100	0.20	0.25	0.20	0.25		

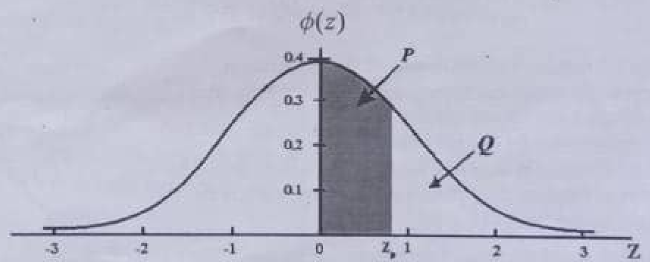
CUMULATIVE NORMAL DISTRIBUTION $P(z)$

											ADD								
z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	0.0000	0040	0080	0120	0160	0199	0239	0279	0319	0359	4	8	12	16	20	24	28	32	36
0.1	0.0398	0438	0478	0517	0557	0596	0636	0675	0714	0753	4	8	12	16	20	24	28	32	36
0.2	0.0793	0832	0871	0910	0948	0987	1026	1064	1103	1141	4	8	12	15	19	22	27	31	35
0.3	0.1179	1217	1255	1293	1331	1368	1406	1443	1480	1517	4	8	11	15	19	22	26	30	34
0.4	0.1554	1591	1628	1664	1700	1736	1772	1808	1844	1879	4	7	11	14	18	22	25	29	32
0.5	0.1915	1950	1985	2019	2054	2088	2123	2157	2190	2224	3	7	10	14	17	21	24	27	31
0.6	0.2257	2291	2324	2357	2389	2422	2454	2486	2517	2549	3	6	10	13	16	19	23	26	29
0.7	0.2580	2611	2642	2673							3	6	9	12	15	19	22	25	28
					2704	2734	2764	2794	2823	2852	3	6	9	12	15	18	21	24	27
0.8	0.2881	2910	2939	2967	2995	3023					3	6	8	11	14	17	20	22	25
							3051	3078	3106	3133	3	5	8	11	13	16	19	22	24
0.9	0.3159	3186	3212	3238	3264	3289					3	5	8	10	13	16	18	21	23
							3315	3340	3365	3389	2	5	7	10	12	15	17	20	22
1.0	0.3413	3438	3461	3485	3508						2	5	7	10	12	14	17	19	22
						3531	3554	3577	3599	3621	2	4	7	9	11	13	15	18	20
1.1	0.3643	3665	3686	3708							2	4	6	8	11	13	15	17	19
					3729	3749	3770	3790	3810	3830	2	4	6	8	10	12	14	16	18
1.2	0.3849	3869	3888	3907	3925						2	4	6	8	10	11	13	15	17
						3944	3962	3980	3997	4015	2	4	5	7	9	11	13	14	16
1.3	0.4032	4049	4066	4082	4099	4115	4131	4147	4162	4177	2	3	5	6	8	10	11	13	14
1.4	0.4192	4207	4222	4236	4251	4265	4279	4292	4306	4319	1	3	4	6	7	8	10	11	13
1.5	0.4332	4345	4357	4370	4382	4394	4406	4418	4429	4441	1	2	4	5	6	7	8	10	11
1.6	0.4452	4463	4474	4484	4495	4505	4515	4525	4535	4545	1	2	3	4	5	6	7	8	9
1.7	0.4554	4564	4573	4582	4591	4599	4608	4616	4625	4633	1	2	3	3	4	5	6	7	8
1.8	0.4641	4649	4656	4664	4671	4678	4686	4693	4699	4706	1	1	2	3	4	4	5	6	6
1.9	0.4713	4719	4726	4732	4738	4744	4750	4756	4761	4767	1	1	2	2	3	4	4	5	5
2.0	0.4772	4778	4783	4788	4793	4798	4803	4808	4812	4817	0	1	1	2	2	3	3	4	4
2.1	0.4821	4826	4830	4834	4838	4842	4846	4850	4854	4957	0	1	1	2	2	2	3	3	4
2.2	0.4861	4864	4868	4871	4875	4878	4881	4884	4887	4890	0	1	1	1	2	2	2	3	3
2.3	0.4893	4896	4898	4901	4904	4906	4909	4911	4913	4916	0	0	1	1	1	2	2	2	2
2.4	0.4918	4920	4922	4925	4927	4929	4931	4932	4934	4936	0	0	1	1	1	1	1	2	2
2.5	0.4938	4940	4941	4943	4945	4946	4948	4949	4951	4952									
2.6	0.4953	4955	4956	4957	4959	4960	4961	4962	4963	4964									
2.7	0.4965	4966	4967	4968	4969	4970	4971	4972	4973	4974									
2.8	0.4974	4975	4976	4977	4977	4978	4979	4979	4980	4981									
2.9	0.4981	4982	4982	4983	4984	4984	4985	4985	4986	4986									
3.0	0.4987	4990	4993	4995	4997	4998	4998	4999	4999	5000									

The table gives $P(z) = \int_0^z \phi(z)dz$

If the random variable Z is distributed as the standard normal distribution N(0,1) then:

1. $P(o < Z < z_p) = P(\text{Shaded Area})$
2. $P(Z > Z_p) = Q = \frac{1}{2} - P$
3. $P(Z > |Z_p|) = 1 - 2P = 2Q$



CUMULATIVE BINOMIAL PROBABILITY (DISTRIBUTION) $\sum_{i \geq r} p_i$

n	r	x = 0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	1	0.0199	0975	1900	2775	3600	4375	5100	5775	6400	6975	7500
	2	0.0001	0025	0100	0225	0400	0625	0900	1225	1600	2025	2500
3	1	0.0297	1426	2710	3859	4880	5781	6570	7254	7840	8336	8750
	2	0.0003	0072	0280	0608	1040	1562	2160	2818	3520	4252	5000
	3		0001	0010	0034	0080	0156	0270	0429	0640	0911	1250
4	1	0.0394	1855	3439	4780	5904	6836	7599	8215	8704	9085	9375
	2	0.0006	0140	0523	1095	1808	2617	3483	4370	5248	6090	6875
	3		0005	0037	0120	0272	0508	0837	1265	1792	2415	3125
	4			0001	0005	0016	0039	0081	0150	0256	0410	0625
5	1	0.0490	2262	4095	5563	6723	7627	8319	8840	9222	9497	9688
	2	0.0010	0226	0815	1648	2627	3672	4718	5716	6630	7438	8125
	3		0012	0086	0266	0579	1035	1631	2352	3174	4069	5000
	4			0005	0022	0067	0156	0308	0540	0870	1312	1875
	5				0001	0003	0010	0024	0053	0102	0185	0312
6	1	0.0585	2649	4686	6229	7379	8220	8824	9246	9533	9723	9844
	2	0.0015	0328	1143	2235	3446	4661	5798	6809	7667	8364	8906
	3		0022	0158	0473	0989	1694	2557	3529	4557	5585	6562
	4		0001	0013	0059	0170	0376	0705	1174	1792	2553	3438
	5			0001	0004	0016	0046	0109	0223	0410	0692	1094
	6					0001	0002	0007	0018	0041	0083	0156
7	1	0.0679	3017	5217	6794	7903	8665	9176	9510	9720	9848	9922
	2	0.0020	0444	1497	2834	4233	5551	6706	7662	8414	8976	9375
	3		0038	0257	0738	1480	2436	3529	4677	5801	6836	7734
	4		0002	0027	0121	0333	0706	1260	1998	2898	3917	5000
	5			0002	0012	0047	0129	0288	0556	0963	1529	2266
	6				0001	0004	0013	0038	0090	0188	0357	0625
	7						0001	0002	0006	0016	0037	0078
8	1	0.0773	3366	5695	7275	8322	8999	9424	9681	9832	9916	9961
	2	0.0027	0572	1869	3428	4967	6329	7447	8309	8936	9368	9648
	3	0.0001	0058	0381	1052	2031	3215	4482	5722	6846	7799	8555
	4		0004	0050	0214	0563	1138	1941	2936	4059	5230	6367
	5			0004	0029	0104	0273	0580	1061	1737	2604	3633
	6				0002	0012	0042	0113	0253	0498	0885	1445
	7					0001	0004	0013	0036	0085	0181	0352
	8							0001	0002	0007	0017	0039
9	1	0.0865	3698	6126	7684	8658	9249	9596	9793	9899	9954	9980
	2	0.0034	0712	2252	4005	5638	6997	8040	8789	9295	9615	9805
	3	0.0001	0084	0530	1409	2618	3993	5372	6627	7682	8505	9102
	4		0006	0083	0339	0856	1657	2703	3911	5174	6386	7461
	5			0009	0056	0196	0489	0988	1717	2666	3786	5000
	6			0001	0006	0031	0100	0253	0536	0994	1658	2539
	7					0003	0013	0043	0112	0250	0498	0898
	8						0001	0004	0014	0038	0091	0195
	9								0001	0003	0008	0020
10	1	0.0956	4013	6513	8031	8926	9437	9718	9865	9940	9975	9990
	2	0.0043	0861	2639	4557	6242	7560	8507	9140	9536	9767	9893
	3	0.0001	0115	0702	1798	3222	4744	6172	7384	8327	9004	9453
	4		0010	0128	0500	1209	2241	3504	4862	6177	7340	8281
	5		0001	0016	0099	0328	0781	1503	2485	3669	4956	6230
	6			0001	0014	0064	0197	0473	0949	1662	2616	3770
	7				0001	0009	0035	0106	0260	0548	1020	1719
	8					0001	0004	0016	0048	0123	0274	0547
	9							0001	0005	0017	0045	0107
	10									0001	0003	0010
11	1	0.1047	4312	6862	8327	9141	9578	9802	9912	9964	9986	9995
	2	0.0052	1019	3026	5078	6779	8029	8870	9394	9698	9861	9941
	3	0.0002	0152	0896	2212	3826	5448	6873	7999	8811	9348	9673
	4		0016	0185	0694	1611	2867	4304	5744	7037	8089	8867
	5		0001	0028	0159	0504	1146	2103	3317	4672	6029	7256

CUMULATIVE BINOMIAL PROBABILITY (DISTRIBUTION)

n	r	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
11	6			0003	0027	0117	0343	0782	1487	2465	3669	5000
	7				0003	0020	0076	0216	0501	0994	1738	2744
	8					0002	0012	0043	0122	0293	0610	1133
	9						0001	0006	0020	0059	0148	0327
	10								0002	0007	0022	0059
	11										0002	0005
12	1	0.1136	4596	7176	8578	9313	9683	9862	9943	9978	9992	9998
	2	0.0062	1184	3410	5565	7251	8416	9150	9576	9804	9917	9968
	3	0.0002	0196	1109	2642	4417	6093	7472	8487	9166	9579	9807
	4		0022	0256	0922	2054	3512	5075	6533	7747	8655	9270
	5		0002	0043	0239	0726	1576	2763	4167	5618	6956	8062
	6			0005	0046	0194	0544	1178	2127	3348	4731	6128
	7			0001	0007	0039	0143	0386	0846	1582	2607	3872
	8				0001	0006	0028	0095	0255	0573	1117	1938
	9					0001	0004	0017	0056	0153	0356	0730
	10							0002	0008	0028	0079	0193
	11								0001	0003	0011	0032
	12										0001	0002
15	1	0.1399	5367	7941	9126	9648	9866	9953	9984	9995	9999	1.0000
	2	0.0096	1710	4510	6814	8329	9198	9647	9858	9948	9983	9995
	3	0.0004	0362	1841	3958	6020	7639	8732	9383	9729	9893	9963
	4		0055	0556	1773	3518	5387	7031	8273	9095	9576	9824
	5		0006	0127	0617	1642	3135	4845	6481	7827	8796	9408
	6		0001	0022	0168	0611	1484	2784	4357	5968	7392	8491
	7			0003	0036	0181	0566	1311	2452	3902	5478	6964
	8				0006	0042	0173	0500	1132	2131	3465	5000
	9				0001	0008	0042	0152	0422	0950	1818	3036
	10					0001	0008	0037	0124	0338	0769	1509
	11						0001	0007	0028	0093	0255	0592
	12							0001	0005	0019	0063	0176
	13								0001	0003	0011	0037
	14										0001	0005
20	1	0.1821	6415	8784	9612	9885	9968	9992	9998	1.0000	1.0000	1.0000
	2	0.0169	2642	6083	8244	9308	9757	9924	9979	9995	9999	1.0000
	3	0.0010	0755	3231	5951	7939	9087	9645	9879	9964	9991	9998
	4		0159	1330	3523	5886	7748	8929	9556	9840	9951	9987
	5		0026	0432	1702	3704	5852	7625	8818	9490	9811	9941
	6		0003	0113	0673	1958	3828	5836	7546	8744	9447	9793
	7			0024	0219	0867	2142	3920	5834	7500	8701	9423
	8			0004	0059	0321	1018	2277	3990	5841	7480	8684
	9			0001	0013	0100	0409	1133	2376	4044	5857	7483
	10				0002	0026	0139	0480	1218	2447	4086	5881
	11					0006	0039	0171	0532	1275	2493	4119
	12					0001	0009	0051	0196	0565	1308	2517
	13						0002	0013	0060	0210	0580	1316
	14							0003	0015	0065	0214	0577
	15								0003	0016	0064	0207
	16									0003	0015	0059
	17										0003	0013
	18											0002

To obtain $p(i \le r)$ use: $p(i \le r) = 1 - p(i \ge r+1)$

Where a space in the table is empty the probability is less than 0.00005.

SENIOR SIX MATHEMATICS SEMINAR TO BE HELD ON 27TH FEBRUARY 2021 ONLINE SEMINAR THROUGH ZOOM.

APPLIED MATHEMATICS P425/2

Instructions:

As a school, please select several questions and prepare presentations in each. We shall book the questions of choice through the WhatsApp group and on the seminar day.

STATISTICS/PROBABILITY:

1. Box A contains 4 red and 5 blue pens, while box B contains 4 red and 4 blue pens. It is known that box A is three times likely to be selected as box B. A box is selected at random and two pens are drawn from it without replacement.

   (a) Find the probability of picking pens of different colours.

   (b) Construct a probability distribution for the number of blue pens that are drawn. Hence calculate the variance of the number of pens drawn from the box.

2. The following table gives the distribution of the interest paid to 500 shareholders of Moringa Growers' Association at the end of 2000.

Interest x 1000 USh	25 -	30 -	40 -	60 -	80 -	110 -	120- < 130
No. of share holders	17	55	142	153	93	20	20

i) Draw a histogram to illustrate this data.

ii) Find the average interest each shareholder receives.

(iii) Calculate the standard deviation of the distribution above.

(iv) Using a cumulative frequency curve or otherwise, find the range of the interest obtained by the middle 80% of the shareholders.

(b) A four-man team is to be selected from three women and 4 men,

(a) What is the probability that (i) the women will form the majority?

(ii) at least 3 men will be on the committee?

(b) If the group contains a couple that insists on being on the committee together repeat question (a)(i) and (ii) above.

3. In a machine manufacturing company, 85% of the nails made are approximately within the set tolerance limits. If a random sample of 200 nails is taken, find the probability that;

   (i) exactly 33 are outside the tolerance limits.

   (ii) between 21 and 27 nails, inclusive, will be outside the tolerance limits.

4. The continuous random variable X has a probability function,

$$f(x)=\begin{cases} k(x+2); & -1 < x < 0 \\ 2k; & 0 \le x \le 1 \\ \frac{k(5-x)}{2}; & 1 < x \le 3 \\ 0, & elswhere \end{cases}$$

   where k is a constant.

   (a) Sketch f(x) and hence find the constant, k

   (b) median

   (c) $P(\frac{1}{2} < x < 2)$

5. (a) The weights of ball bearings are normally distributed with mean 25gram and standard deviation 4 grams. If a random sample of 16 ball bearings is taken, find the;

   (i) Probability that the mean of the sample is between 24.12 grams and 26.73 grams.

   (ii) Interquartile range.

   (c) A random sample of 120 girls taken from a normally distributed population of girls school gave a mean age of 16.5 and variance of 18. Determine the 97% confidence interval for the mean age of all the school girls.

MECHANICS:

6. (a) A car traveling at 54 kmh^{-1} is brought to rest with uniform retardation in 5 seconds. Find its retardation in ms^{-2} and distance it travelled in this time

   (b) A cyclist was timed between successive trading centres P; Q and R, each 2 km a part. It took $\frac{5}{3}$ minutes to travel from P to Q and 2.5 minutes from Q to R. Find;

   (i) the acceleration

   (ii) the velocity with which the cyclist passes point P

   (iii) How much further the cyclist will travel before coming to rest if the acceleration remains uniform.

7. A bullet is fired from a point P which is at the top of a hill 50m above the ground. The speed with which the bullet is fired is 140ms^{-1} and it hits the ground at a point Q which is at a horizontal distance 200m from the foot of the bill. Find the,

   (i) two possible values of angle of projection

   (ii) two possible times of flight.

   (iii) angle with which the bullet hits the grounds.

8. Two smooth inclined planes meet at right angles, the inclination of one to the horizontal being 30° and the other being 60°. Bodies of masses 2kg and 4kg lie on the respective planes, and the two masses are joined by a light inextensible string passing over a smooth fixed pulley at the intersection of the planes.

   (i) If the masses are released from rest, find the acceleration of the masses and tension of the string.

   (ii) If the surface with 4kg mass experiences a frictional force of 0.25N, find the new acceleration of the masses when the system is set from rest.

9. (a) A pump draws water from a tank and issues it at a speed of 8 ms^{-1} from the end of a pipe of cross-sectional area of 0.01m^2 situated 10m above the level from which the water is drawn. Find the rate at which the pump is working (take density of water = 1000kgm^{-3})

(b) A car of mass 800kg is pulling a trailer of mass 200kg up a hill inclined at an angle $Sin^{-1}\left(\frac{1}{14}\right)$ to the horizontal. When the total force exerted by the engine is 1000N the car and the trailer move up the hill at a steady speed.

(i) Find the total frictional resistance to the motion of the car and trailer during this motion

(ii) If the frictional resistance on the car is 280N, find the tension in the coupling between the car and the trailer.

10. A particle A initially at the point with position vector $2\mathbf{i} - 5\mathbf{j} + \mathbf{k}$ km is moving with a constant velocity of $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ kmh^{-1}. At the same instant, a particle B at the point $(3, 3, 2)$is moving with a constant velocity of $3\mathbf{i} - 2\mathbf{k}$ kmh^{-1}. Find the:

(i) relative velocity of particle A to B.

(ii) relative displacement of particle A to particle B at any instant.

(iii) shortest distance between the two particles in their subsequent motion.

11. A body moving with an acceleration $(2e^{4t}\boldsymbol{i} - 3Cost\,\boldsymbol{j} + 4Sin2t\,\boldsymbol{k})$ m/s^2 is initially located at a point whose position vector is $(\boldsymbol{i} - \boldsymbol{2j} + \boldsymbol{k})m$ and has a velocity $(\boldsymbol{i} - \boldsymbol{2j} + \boldsymbol{2k})\ m/s$ at time t = 0.

Find the (a) velocity of the body at time t= $\frac{\pi}{2}$s

(b) displacement of the body at any time t =1s.

12. A non-uniform ladder AB whose centre of gravity is 2m from end A is of length 6m and weight, **W**. The ladder is inclined at an angle θ to the vertical with its end B against a rough vertical wall and end A on a rough horizontal ground with which the coefficients of friction at each point of contact is μ. If the ladder is about to slip when a man of weight **5W** ascends two-thirds of the way up the ladder, show that $\boldsymbol{tan\theta = \frac{18\mu}{11-7\mu^2}}$

13. (a) ABCD is a square of side 2m. Forces of magnitudes 3N, 5N, 7N and 2N act a long sides DA, AB, BC and CD respectively. Calculate the:

(i) magnitude of the resultant of the forces and the angle made by the resultant with AD.

(ii) sum of the moments of the forces about A

(iii) distance from A of the point where the line of action of the resultant of the forces cuts DA produced.

(iv) equation of the line of action of the resultant.

14. (a) A smooth bead of mass 0.2 kg is threaded on a smooth circular wire of radius r metres which is held in a vertical plane. If the bead is projected from the lowest point on the circle with speed $\sqrt{3rg}$. Find the;

  (a) speed of the bead when it has gone one sixth of the way round the circle.

  (b) force exerted on the bead by the wire at this point.

NUMERICAL METHODS:

15. (a) Give X = 12.6955

i) Truncate to 2 significant figures ii) Round to 2 decimal places

b) Given A = 7.684, B = 0.31, Write down the maximum possible errors in A and B. Hence, find; (i) absolute error in the quotient $\frac{A + B}{AB}$

(ii) Interval within which the quotient in b (i) above lies.

c) The quantities **p** and **q** were measured with errors e_1 and $e_2$respectively. Show that the maximum relative error in $\frac{p}{\sqrt{q}}$ is given by $\left|\frac{e_1}{p}\right| + \frac{1}{2}\left|\frac{e_2}{q}\right|$

Hence find the range within which $\frac{2.35}{\sqrt{5.1}}$ lies

16. a) A school canteen started its operation with a capital of 2 million shillings. At the end of a certain term its profits in soft drinks and foodstuff were 0.2 million and 0.45 million respectively. There were possible errors of 6.5% and 8.2% in the sales

respectively. Find the maximum and minimum values of the sales as a percentage of the capital.

b) The charges of sending parcels by a certain, distributing company depends on the weights, 500g, 1kg, 1.5kg, 2kg and 5kg the charges are 750/=, 1000/=, 2000/=. 3500/=, and 5500/= respectively. Estimate;

i) What would the distributor charge for a parcel of weight 2.7kg?

ii) If the sender pays 6200/=, what is the weight of the parcel?

17. Show that the Newton – Raphson formula for solving the transcendental equation

$\sin x - \frac{1}{2} = 0$ is given by $x_{n+1} = \dfrac{x_n \cos x_n - \sin x_n + \frac{1}{2}}{\cos x_n}$

Show also that one of the roots of the equation lies between 1.5 and 2. Hence find the root to two decimal places.

18. (a) Use the trapezium rule with 6 ordinates to find $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos x}\,dx$ correct to three decimal places.

(b) Calculate the percentage error made in the approximation in (a) above.

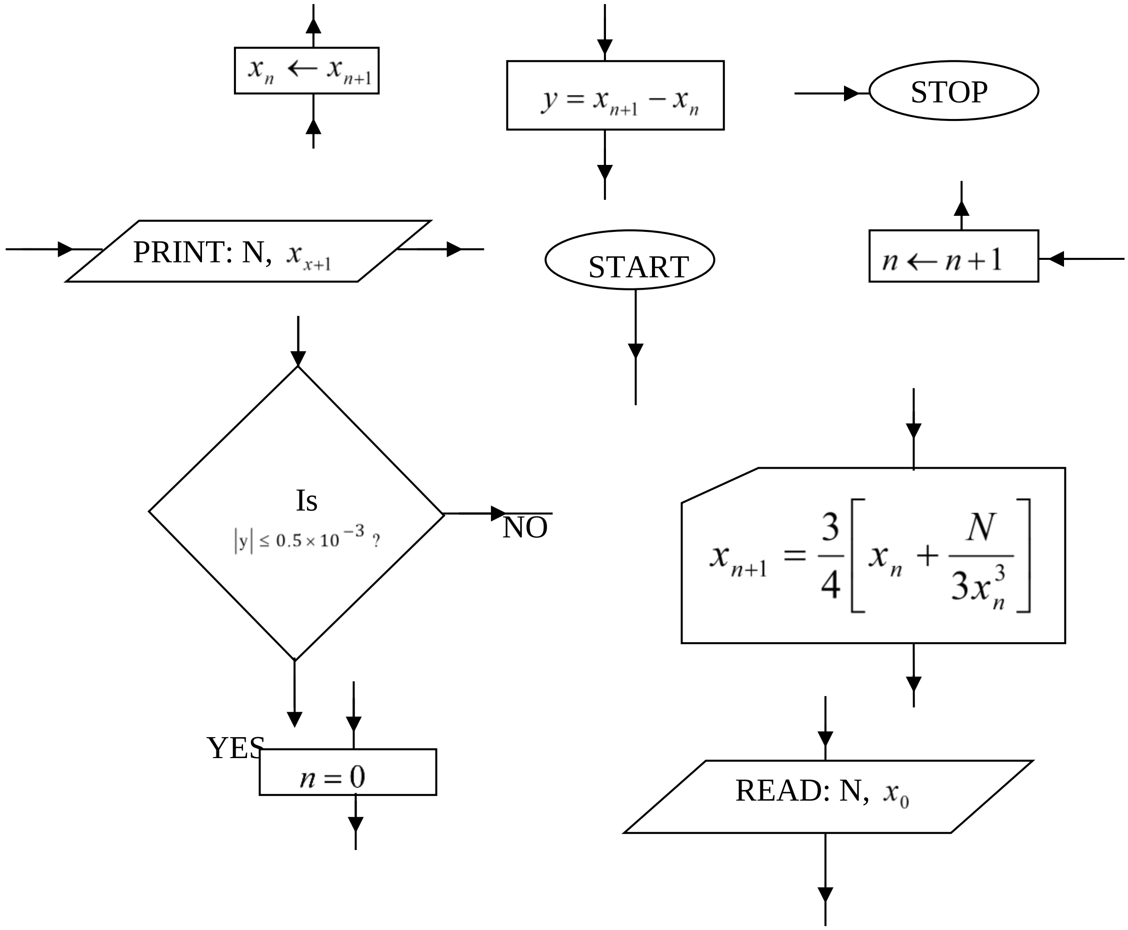
19. Two iterative formulae A and B shown below are used to solve the equation f(x) =0. Only one of these re-arrangements provides a formula which converges to the root.

A	B
$x_{n+1} = \frac{1}{3}(x_n^3 + 1)$	$x_{n+1} = \frac{3x_n - 1}{x_n^2}$

(i) Determine the equation whose root is being sought.

(ii) Starting with $x_0 = 0.2$, use the most suitable formula to find the root of the equation f(x) =0 correct to 2 decimal places.

20. Given below are elements of a flow chart not rearranged in order



(i) Re-arrange the elements to form a logical flow chat. State the purpose of the flow chart.

(ii) Perform a dry run of your flow chart, taking N= 187.42 and $x_0 = 3.0$. Give your answer correct to three decimal places.

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v. The teachers will also make any additional comments(5min)

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Topic: A-LEVEL MATHEMATICS VIRTUAL SEMINAR
Time: Feb 27, 2021 08:30 AM Nairobi

Join Zoom Meeting
https://us02web.zoom.us/j/81410189011?pwd=Rit6YzJCNzA3RzdMOFMrNnJyTUdyZz09

Meeting ID: 814 1018 9011
Passcode: 940152

SENIOR SIX MATHEMATICS SEMINAR TO BE HELD ON 27TH FEBRUARY 2021 ONLINE SEMINAR THROUGH ZOOM.

PURE MATHEMATICS P425/1

Instructions:

As a school, please select several questions and prepare presentations in each. We shall book the questions of choice through the WhatsApp group and on the seminar day.

VECTORS

1. The lines l_1 and l_2 have equations $l_1: r = 6i + 5j + 4k + \lambda(i + j + k)$ and $l_2: r = 6i + 5j + 4k + \mu(4i + 6j + k)$.
   (i) Find the cartesian of the plane Π containing l_1 and l_2.
   (ii) Find the position vector of the foot of the perpendicular from the point with position vector $i + 10j + 3k$ to the plane Π.
   (iii) The line l_3 has equation $r = i + 10j + 3k + t(2i - 3j + k)$. Find the shortest distance between l_1 and l_3.
2. The lines l_1 and l_2 have vector equations $r = 4i - 2j + \lambda(2i + j - 4k)$ and $r = 4i - 5j + 2k + \mu(i - j - k)$ respectively.
   (i) Find the angle between the lines l_1 and l_2.
   (ii) Show that l_1 and l_2 intersect and find the point of intersection.
   (iii) Find the perpendicular distance from the point P whose position vector is $3i - 5j + 6k$ to the plane containing l_1 and l_2.

ANALYSIS/CALCULUS:

1. The curve C has equation $y = \frac{x(x+1)}{(x-1)^2}$.
   Find
   (a) (i) the asymptotes of C.
   (ii) the point P where the curve C intersects with the asymptotes.
   (iii) the nature of the stationary points of C if they exist.
   (b) Draw the sketch of curve C.
2. (i) Apply a suitable substitution to simplify and evaluate
   $\int_0^1 \frac{\sqrt{x}}{2-\sqrt{x}} dx$

Contact- Mr.Senjobe Elsam , Gayaza High School-0704899034
Mr. Sekayita George, Gayaza High School- 0754230088
Mr. Khalid Muhamood, Hope boarding secondary school, 0754857296

(ii) Evaluate $\int_0^{\frac{\pi}{2}}(x-1)\cos 2x\, dx$

3. (a) Find

$\int \frac{8^{\sqrt{x}}}{\sqrt{x}}\, dx$

(c) The Area enclosed by the curve $y = e^x$, the y-axis and the line $y = 4$ is rotated through 360^o about the x-axis. Prove that the volume, V of the solid generated is $V = e^4 \ln 4 - \frac{15\pi}{2}$.

4. Express $\frac{2x^2-9x\pm 6}{x(x^2-x-6)}$ in partial fractions.

Hence, evaluate $\int_4^6 \frac{2x^2-9x\pm 6}{x(x^2-x-6)}\, dx$

5. A pond covers an area of $300\ m^2$. A specimen of pondweed grows on the surface of the pond. At time *t* days after the weed is first discovered, it covers an area of $A\ m^2$.
The area of the pond covered by the weed increases at a rate which is proportional to the square root of the area of the pond already covered by the weed. Initially, the area covered by the weed is $0.25\ m^2$ and its rate of growth per day is $1\ m^2$.
(i) Form a differential equation to show the relationship between A and *t* .
(ii) Deduce to the nearest day, the time taken for the pond's surface area to be completely covered by the weed.

GEOMETRY:

1. (a) Three circles of radius $\frac{3}{2}$ have their centres at the points (3,0), (3,5) and (0,5). Prove that the equation of the circle which cuts all three orthogonally is $4x^2 + 4y^2 - 12x - 20y + 9 = 0$.
(b) Find the equation and radius of a circle passing through the points $A(0,1)$, $B(0,4)$ and $C(2,5)$.

2. (a) Prove that the equations of the tangents to the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ with gradient m are the equations $mx - y \pm \sqrt{a^2m^2 + b^2} = 0$.

(b) Find the locus of the point of intersection of the tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ that are perpendicular to each other.

3. Find the equation of the tangent and the normal to the curve $xy = c^2$ at the point $P\left(ct,\ {}^{c}/_{t}\right)$. Given that the normal at P meets the curve again at Q, find the coordinates of

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Q. If the tangent at P meets the y-axis at R, find the equation of the locus of M, the mid-point of PR.

TRIGONOMETRY:

1. Solve the equation: $\cos^{-1} 2x - \cos^{-1} x = \dfrac{\pi}{3}$.

   b) Find the maximum and minimum values of the function $\dfrac{1}{3+\sin x - 2\cos x}$ stating clearly the values of x.

2. Given that $p = 2\cos\theta + 3\cos 2\theta$ and $q = 2\sin\theta + 3\sin 2\theta$:

   i) find the greatest and least values of $p^2 + q^2$.

   ii) given that $p^2 + q^2 = 19$, find θ for $0^o \le \theta \le 90^o$.

   iii) Show that $pq = -\dfrac{5\sqrt{3}}{4}$.

3. (a) Show that in any triangle ABC, if $2s = a + b + c$,

   $1 - \tan\frac{1}{2}A\ \tan\frac{1}{2}B = \frac{c}{s}$.

   (b) Prove that $4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right) = \frac{\pi}{4}$.

ALGEBRA

1. (a) The function $\boldsymbol{f = x^3 + px^2 - 5x + q}$ has a factor $(x-2)$ and a value of 5 when $x = -3$. Find the values of p and q.

   (b) The roots of the equation $\boldsymbol{x^2 + x + 1 = 0}$ are α and β. Form the quadratic equation whose roots are $\boldsymbol{\alpha\beta}$ and $\boldsymbol{\alpha}$.

   (c) Simplify: $\boldsymbol{\frac{\sqrt{3}-2}{2\sqrt{3}+3}}$ in the form $\boldsymbol{p + q\sqrt{3}}$ where p, q are rational numbers.

2. (a) Solve the equation $\log_2 x - \log_8 x = 2$.

   b) Solve $\log_x 5 + 4\log_5 x = \log_4 256$.

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c)Solve for x for which $2(3^{2x}) - 5(3^x) + 2 = 0$.

(d) Find the solution of the inequlity $\frac{x+1}{x-1} < \frac{x+3}{x+2}$.

3. Simplify:

(a). $\frac{x^2(x^2+1)^{-\frac{1}{2}} - (x^2+1)^{\frac{1}{2}}}{x^2}$. (b). $\frac{(8)^{\frac{1}{6}} \; X \; (4)^{\frac{1}{3}}}{(32)^{\frac{1}{6}} \; X \; (16)^{\frac{1}{12}}}$ (c). $\left(\frac{16}{81}\right)^{-\frac{1}{4}}$

CURVE SKETCHING

1. Sketch the curve $y = \frac{4(x-3)}{x(x+2)}$ by investigating clearly the horizontal region of inexistence
2. Sketch the curve $y = \frac{4x^2}{(x^2+1)(x-2)}$
3. By clearly stating the turning point and all the asymptotes, ketch the curve $y = \frac{x^3}{x^2-4}$
4. Sketch the curve $y = \frac{3x+3}{x(3-x)}$ clearly find the turning points, asymptotes and the horizontal region where the curve does not exist

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OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL PURE MATHEMATICS SEMINAR QUESTIONS 2024

ORGANISED ON SATURDAY 05TH OCTOBER 2024.

ALGEBRA

1. (a) The sum of n terms of a particular series is given by $S_n = 17n - 3n^2$;
(i) Find an expression for the n^{th} term of the series.
(ii) Show that the series is an Arithmetic progression.

(b) A student deposits shs. 1,200,000 once into her savings account on which an interest of 8% is compounded per annum. After how many years will her balance exceed shs, 200,000?

(c) A piece of land of area $50{,}100m^2$ is divided in such a way that the areas of the plots are in an Arithmetic progression (AP). If the area of the smallest and the largest plots are $2m^2$ and $1000m^2$ respectively, find the;

(i) Number of plots in the piece of land.
(ii) Total area of the first 13 plots to the nearest square metres.

2. (a) Solve the inequality $\frac{x+3}{x-2} \geq \frac{x+1}{x-2}$

(b) Given the curve, $y = \frac{(x-1)(x-4)}{(x-5)}$
(i) Find the range of values of y for which the curve doesnot lie and hence deduce the coordinates of the turning points.
(ii) Show that $y = x$ is an asymptote and state the other asymptote
(iii) Sketch the curve.

3. (a) Solve for x; $64x^{\frac{2}{3}} + x^{\frac{-2}{3}} = 20$
(b) Find the ratio of the coefficient of x^7 to that of x^8 in the expression of $\left(3x + \frac{2}{3}\right)^{17}$
(c)(i) Expand $(1 + x)^{-2}$ in descending powers of x including the term in x^{-4}
(ii) If $x = 9$, find the % error in using the first two terms of the expression in c(i) above.

4. (a) Given that W and Z are two complex numbers, solve the simultaneous equations;
$$3Z + W = 9 + 11i$$
$$iW - z = -8 - 2i$$
(b) Use Demoivre's theorem to simplify; $\frac{[\sqrt{3}(cos\theta + isin\theta)]^8}{[3cos2\theta + 3isin2\theta]^3}$
(c) If $(1 + 3i)z_1 = 5(1 + i)$, show that the locus of $|z - z_1| = |z_1|$ where Z is a complex number is a circle and find its Centre and radius
(d) Given that the factors $(x - 1)$ and $(x + 1)$ are factors of the polynomial, $f(x) = ax^4 + 7x^3 + x^2 + bx - 3$, find the values of the constants a and b. Hence, find the set for real values of x for which $f(x) > 0$

TRIGONOMETRY

5. (a) Prove that $\tan(\theta + 60^0)\tan(\theta - 60^0) = \frac{tan^2\theta - 3}{1 - 3tan^2\theta}$

(b) Show that $-\sqrt{5} \leq cosx + 2sinx \leq \sqrt{5}$

(c) Express $10cosxsinx + 12cos2x$ in the form $Rsin(2x + \beta)$, where R is positive and β is an acute angle. Hence find the maximum and minimum values of $10cosxsinx + 12cos2x$ and state clearly the values of x when they occur for $0^0 \leq x \leq 360^0$.

6. (a) Solve the equation: $\frac{4sin^2\theta}{cosec\theta} + \frac{3}{cosec^2\theta sec\theta} = sin^2\theta$ for $0^0 \leq \theta \leq 360^0$

(b) (i) Prove that $\frac{sin2A+cos2A+1}{sin2A+cos2A-1} = \frac{\tan(45^0+A)}{tanA}$

(ii) Show that $\frac{sin\theta cos2\theta+sin3\theta cos6\theta}{sin\theta sin2\theta+sin3\theta sin6\theta} = cot5\theta$

(c) Show that $\frac{sin\theta}{1-cos\theta} = cot\frac{\theta}{2}$. Hence solve $tan\frac{\theta}{2} = \sqrt{3}sin\theta$ for $0^0 \leq \theta \leq 180^0$

7. (a) Given that X, Y, Z are angles of a triangle. Prove that $\tan\left(\frac{X-Y}{2}\right) = \left(\frac{x-y}{x+y}\right) cot\left(\frac{Z}{2}\right)$, hence solve the triangle if $x = 9cm, y = 5.7cm$ and $z = 57^0$

(b) Prove that $Sin[2sin^{-1}(x) + cos^{-1}(x)] = \sqrt{1-x^2}$

(c) Solve the equation; $2\sin(60^0 - x) = \sqrt{2}cos(135^0 + x) + 1$ for $-180^0 \leq x \leq 180^0$

8. (a) If $tanx = \frac{7}{24}$, and $cosy = \frac{-4}{5}$ where x is reflex and y is obtuse, find without using tables or calculators the value of $sin(x + y)$

(b) In a triangle ABC, $\overline{AB} = 10cm, \overline{BC} = 17cm$ and $\overline{AC} = 21cm$ calculate the angle BAC.

(c) Solve the equation $sin3x + sin7x = sin5x$ for $0^0 \leq x \leq 90^0$

(d) (i) Given that $2A + B = 135$ show that $tanB = \frac{tan^2A-2tanA-1}{1-2tanA-tan^2A}$

(ii) If α is an acute angle and $tan\alpha = \frac{4}{3}$, show that $4sin(\theta + \alpha) + 3cos(\theta + \alpha) = 5cos\theta$. Hence solve for θ the equation $4sin(\theta + \alpha) + 3cos(\theta + \alpha) = \frac{\sqrt{300}}{4}$ for $-180^0 \leq \theta \leq 180^0$

ANALYSIS

9. (a) The point (2,1) lies on the curve $Ax^2 + By^2 = 11$ where A and B are constants.

If the gradient of the curve at the point is 6. Find the values of A and B.

(b) One side of a rectangle is three times the other. If the perimeter increases by 2%. What is the percentage increase in the area?

(c) A rectangular box without a lid is made from a thin cardboard. The sides of the base are $2xcm$ and $3xcm$ and the height of the box is hcm. If the total surface area is $200cm^2$, show that $h = \left(\frac{20}{x} - \frac{3x}{5}\right) cm$. And hence find the dimensions of the box to give maximum volume.

10. (a) If $y = \frac{\cos x}{x^2}$, Prove that; $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (2 + x^2)y = 0$

(b) Given the parametric equations $x = 3 + 4\cos\alpha$, $y = 5 - 8\sin\alpha$. Find $\frac{d^2y}{dx^2}$

(c) A curve is defined by the parametric equations $x = t^2 - t$, $y = 3t + 4$. Find the equation of the tangent to the curve at (2,10)

(d) Using calculus of small changes, Show that $\cos 44.6^0 = \frac{\sqrt{2}}{2}\left(\frac{900+2\pi}{900}\right)$

11. (a). Show that $\int_1^{10} x\log x^2 dx = 2\left(50 - \frac{99}{4\ln 10}\right)$

(b) Express $\frac{x^3+9x^2+28x+28}{(x+3)^2}$ into partial fractions, hence or otherwise show that;
$\int_0^1 \frac{x^3+9x^2+28x+28}{(x+3)^2} dx = \frac{1}{3}\left(10 + \ln\frac{4}{3}\right)$

(c) Find the integrals; (i) $\int \ln\left(\frac{2}{x}\right) dx$ (ii) $\int (x\cos x)^2 dx$ (iii) $\int \frac{x}{\sqrt{1-3x}} dx$

12.(a) The pressure in an engine cylinder is given by; $P = 8000[1 - \sin(2\pi t - 3)]Nm^{-1}$ At what time does this pressure reach a maximum and what is the maximum pressure.

(b) Calculate the area enclosed by the curve $y = \sin x$ and the line $y = \frac{1}{2}$, from $x = 0$ to $x = \pi$ and the x-axis.

(c) The area bounded by the curves $y^2 = 32x$ and $y = x^3$ is rotated about the x-axis through one revolution. Show that the volume of the solid of the solid formed is $\frac{320\pi}{7}$ cubic units

(d) Using Maclaurin's theorem, expand $(x + 1)\sin^{-1}(x)$ up to the term in x^2

13. (a) Using the substitution $y = uv$, solve the differential equation $x^2\frac{dy}{dx} = x^2 + xy + y^2$

(b) Given that $\frac{dy}{dx} = e^{-2y}$ and $y = 0$, when $x = 5$, find the value of x when $y = 3$

(c) Solve the differential equation $(1 + x)\frac{dy}{dx} = xy + xe^x$ given that $y(0) = 1$

(d) The rate at which a liquid runs from a container is proportional to the square root of the depth of the opening below the surface of the liquid. A cylindrical petrol storage tank is sunk in the ground with its axis vertical. There is a leak in the tank at an unknown depth. The level of the petrol in the tank originally full is found to drop by 20cm in 1 hour and by 19cm in the next hour. Find the depth at which the leak is located.

VECTORS

14. (a) Point B is the foot of a perpendicular from point $A\ (3,\ 0, -2)$ to the line $\boldsymbol{r}$ where $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

(i) Find the values of λ corresponding to the point B. hence state the coordinates of B.

(ii) Calculate the distance of the point A from the line $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ and write down the vector parametric equation of the plane containing point A and the line $\boldsymbol{r}$

(b) Find the area of a parallelogram of which the given vectors are adjacent sides, $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{j} + \boldsymbol{k}$ respectively.

(c) A and B are points (3,1,1) and (5,2,3) respectively and C is a point on the line $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$. If angle$BAC = 90^0$, find the coordinates of C.

15. (a) Find the coordinates of the point where the line $\frac{x-3}{5} = \frac{3-y}{-2} = \frac{z-4}{3}$ meets the plane $2x - 3y + 7z - 10 = 0$

(b) The vector $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ is perpendicular to the plane containing the line; $\frac{x-3}{-2} = \frac{y+1}{a} = \frac{z-2}{1}$, find the;

(i) Value of a
(ii) Cartesian equation of the plane

(c) Find the perpendicular distance from the point M (4,-3,10) to the line with vector equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

16. (a) Two planes L_1 and L_2 are defined by $3x - 4y + 2z - 5 = 0$ and $\boldsymbol{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ respectively.

Find;
(i) Cartesian equation of plane L_2
(ii) Acute angle between the two planes
(iii) Vector equation of the line of intersection of L_1 and L_2
(b) Given the points L (2,-1, 0), M (4, 7, 6) and N (8, 5,-4). Find the vector equation of the line which joins the midpoint of LM and MN.
(c) Determine the equation of the plane equidistant from the points A (1, 3, 5) and B (2,-4, 4)

17. (a) Find the equation of the line through point A(1,-2,3) perpendicular to the line $\frac{x-5}{2} = \frac{y-2}{1} = \frac{z-1}{3}$
(b) Prove that Points A (-2,0,6) and B(3,-4,5) lie on opposite sides of the plane $2x - y + 3z = 21$
(c) Find the equation of a plane containing points A (1, 1, 1), B (1, 0, 1) and C (3, 2,-1)
(d) Show that the vectors $2i - j + k, i - 3j - 5k$ and $3i - 4j - 4k$ are coplanar
(e) Point R with position vector **r** divides the line segment AB internally in the ratio $\lambda : \mu$, Show that $r = \frac{a\mu + b\lambda}{\lambda + \mu}$ where a and b are position vectors of A and B respectively. Hence find the position vector of point R which divides AB in the ratio 1:2, given that the position vector of A is $\begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$ and that of B is $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

COORDINATE GEOMETRY

18. (a) A line L passes through the point of intersection of the lines $x - 3y - 4 = 0$ and $y + 3x - 2 = 0$. If L is perpendicular to the line $4y + 3x = 0$, determine the equation of the line L.

(b) Variable point $P(x, y)$ moves such that its distance from point$A(3,0)$ is equal to its distance from the line$x + 3 = 0$. Describe the locus of point P.

(c) Calculate the perpendicular distance between the parallel lines $3x + 4y + 10 = 0$ and $3x + 4y - 15 = 0$

(d) Calculate the area of the triangle which has sides given by the equations $2y - x = 1, y + 2x = 8$ and $4y + 3x = 7$

19. (a) The triangle ABC with vertices A(1,-2), B(7,6) and C(9,2), find:

(i) The equations of the perpendicular bisectors of AB and BC.

(ii) The coordinates of the point of intersection of the perpendicular bisectors

(iii) Find the equation of the circle passing through the three points A,B,C of the triangle above.

(b) Show that the circles $x^2 + y^2 - 2ax + c^2 = 0$ and $x^2 + y^2 - 2by - c^2 = 0$ are orthogonal.

(c) Find the length of the tangent to the circle $x^2 + y^2 - 4x + 9 = 0$ from the point (5,7)

20. (a) Determine the vertex, focus, directrix and axis of the parabola $y^2 - 2y - 8x - 17 = 0$ hence sketch the parabola.

(b) The tangents to the parabola $y^2 = 4ax$ at points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ meet at point T, find the coordinates of T.

(c) If $\left(\frac{1}{2}, 2\right)$ is one extremity of a focal chord of the parabola $y^2 = 8x$, find the coordinates of the other extremity.

(d) If $y = mx + c$ is a tangent to the parabola $y^2 = 4ax$, show that $m = \frac{a}{c}$

21. (a) Show that the parametric equations $x = 1 + 4\cos\theta$ and $y = 2 + 3\sin\theta$ represent an ellipse. Hence determine the coordinates of the centre and the lengths of the semi axes

(b) The normal at the point $P(5\cos\theta, 4\sin\theta)$ on an ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ meets the x and y-axes at A and B respectively. Find the mid-point of the line AB

(c)(i) Find the equation of the tangent to the hyperbola whose points are of the parametric form$\left(2t, \frac{2}{t}\right)$.

(ii) Find the equations of the tangents in (i) which are parallel to $y + 4x = 0$

(iii) Determine the distance between the tangents in c(ii).

END

ALGEBRA

1. Express x^2-6x+5 in the form $p+(x-r)^2$ and hence deduce the coordinates of the turning points of $\frac{32}{x^2-6x+5}$ and state the equation of the line of symmetry.

2. If the sum of roots of the equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals, show that $ab^2+bc^2=2a^2c$.

3. Given that $(x+1)^2$ is a factor of the polynomial $2x^4+7x^3+px^2+qx+r$ and has a remainder of 14 when divided by $(x-1)$, find the values of $p,\ q,\ r$.

4. Given a G.P $a+b+\ldots+l$ prove that its sum is $S_n=\frac{bl-a^2}{b-a}$.

5. Given that the equations $x^2+px+q=0$ and $x^2+mx+k=0$ have a common root, show that $(q-k)^2=(m-p)(pk-mq)$.

6. Given that one root of the equation $x^2+px+q=0$ is twice the other, show that $2p^2=9q$, hence, find the values of k, if the equation $x^2-2(k+2)x+(k^2+3k+2)=0$, has one root twice the other.

7. If $\log_e a^3b=u$ and $\log_e ab^2=v$, find a in terms of e, u and v.

8. Solve the equation: $\log_{10} e.In(x^2+1)-2\log_{10} e.Inx=\log_{10} 5$

9. Prove by mathematical induction that for all positive integral values of n, $10^n+3(4^{n+2})+5$ is a multiple of 9.

10i) Prove that the curve $y=\frac{4x^2-10x+7}{(x-1)(x-2)}$ cannot lie in the region $-2\sqrt{3}$ and $2\sqrt{3}$.

ii) Solve the inequality: $\frac{x+1}{2x-3}\le\frac{1}{x-3}$

11. The sum of the first n terms of an A.P is n^2+5n. Find the first three terms of the series.

12. The first term of a geometric progression is A and the sum of the first three terms is $\frac{7}{4}A$.

 i) Show that there are two possible progressions.

 ii) Given that $A = 4$ find the next two terms of each progression.

13. Expand $\frac{1}{\sqrt{1+x}}$ up to the term in x^2 and by letting $x = \frac{1}{4}$, show that $\sqrt{5} \approx \frac{256}{115}$.

14. Express $\left(-1-\sqrt{3}i\right)^6$ in the form $x + iy$.

15. Describe the locus represented by $2|z-2i| = 3|z+1|$ and hence state the centre and radius.

16. If $\frac{(z-i)}{(z-1)}$ is purely imaginary, show that the locus of z is a circle. State the centre and radius.

17. Given $z = 1 + \cos 4\theta + i \sin 4\theta$, find the modulus and argument of z.

18. Solve for n given that ${}^{n}C_4 = 5 \times {}^{n-2}C_3$.

19. Find the square root of $5 + 12i$.

20. If x is sufficiently small the allow any terms in x^5 or higher powers of x to be neglected, show that $(1+x)^6\left(1-2x^3\right)^{10} \cong 1 + 6x + 15x^2 - 105x^4$.

ANALYSIS

21. Evaluate: $\int_{-1}^{-1/2} \frac{(4x+2)}{(x-1)^4(x+2)^4}\,dx$

22. Use small changes to show that $(16.02)^{\frac{1}{4}} \approx 2\frac{1}{1600}$

23. Use $t = \tan\frac{1}{2}x$, to find the value of $\int_0^{\frac{\pi}{2}} \frac{dx}{3 + 5\cos x}$

24. Sketch the curve $y=\frac{4+3x-x^2}{x-8}$, clearly find the nature of the turning points and state their asymptotes.

25. An open cylindrical container is made from a $12cm^2$ metal sheet. Show that the maximum volume of the container is $\frac{8}{\sqrt{\pi}}cm^3$.

26. The area enclosed between the parts of the curves $x^2+y^2=1$ and $4x^2+y^2=4$ for which y is positive is rotated about the $x-$ axis, find the volume of the solid generated.

27. Evaluate : $\int_1^2 (x-1)^2 \; In\; x\, dx$

28. Find Maclaurin's expansion of $y=In\frac{(2-x)^2}{(1+x)^2}$, showing the first three non zero terms, hence, find the approximate value of $2In\frac{1.99}{1.01}$ correct to 3 s.f.

29. Integrate $\int \frac{4x^2+x+1}{(x^3-1)}dx$

30. Find the area enclosed by the curves $y^2=4x$ and $x^2=4y$.

31. Evaluate: $\int_0^{1/4} \cos^{-1} 2x\, dx$

32. Find the equation of the tangent at the point $(1, -1)$ to the curve $y=2-4x^2+x^3$. What are the coordinates of the point where the tangent meets the curve again? Find the equation of the tangent at this point.

33. If $y=\tan\left(2\tan^{-1}\frac{x}{2}\right)$, show that $\frac{dy}{dx}=\frac{4(1+y^2)}{4+x^2}$

34. Differentiate from first principles: $y=\sqrt{\cos x}$ from first principles.

35. A particle is moving in a straight line such that its distance from a fixed point O , $t\,s$ after motion begins is $s=\cos t+\cos 2t\ m$, find:

i) the time when the particle first passes through O.

ii) the velocity of the particle at this instant.

iii) the acceleration when the velocity is zero.

36. Given $\frac{dy}{dx} = 2\cos x\sqrt{y+3}$, find y in terms of x if $y\left(\frac{\pi}{2}\right) = 1$.

37. Solve the d.e $2y(x+1)\frac{dy}{dx} = 4 + y^2$, given that $y = 2$ when $x = 3$ express y in terms of x.

38. The price p of a commodity varies in such a way that the rate of change of price with respect to time t hours is proportional to the shortage $D - S$, where $D = 8 - 2p$ and $S = 2 + p$. If the price at $t = 0$ is $\$5$ and after $t = 2$ hours the price is $\$3$. Find the price of the commodity at any time and determine the price of the commodity as time tends to infinity.

39. Show that $\int_1^2 \frac{2x^3 - 1}{x^2(2x-1)}dx = \frac{3}{2} + \frac{1}{2}In\left(\frac{16}{27}\right)$

TRIGONOMETRY

40. Prove that $(\sin 2\alpha - \sin 2\beta)\tan(\alpha + \beta) = 2(\sin^2\alpha - \sin^2\beta)$.

41. Given $\sin(x+\alpha) = 2\cos(x-\alpha)$, prove that $\tan x = \frac{2 - \tan\alpha}{1 - 2\tan\alpha}$

42. Solve: $\cos(2\theta + 45^o) - \cos(2\theta - 45^o) = 1$, for $0^o \le \theta \le 360^o$.

43. Prove the identity: $\cos^6 x + \sin^6 x = 1 - \frac{3}{4}\sin^2 2x$.

44. The roots of the equation $ax^2 + bx + c = 0$ are $\tan\alpha$ and $\tan\beta$. Express $\sec(\alpha + \beta)$ in terms of a, b, c.

45. If $a = x\cos\theta + y\sin\theta$ and $b = x\sin\theta - y\cos\theta$, prove that $\tan\theta = \frac{bx + ay}{ax - by}$.

46. Solve: $3\tan^3 x - 3\tan^2 x = \tan x - 1$ for $0^o \le x \le \pi$.

47. Prove that $\cos^5 x = \dfrac{\cos 5x + 5\cos 3x + 10\cos x}{16}$.

48. Find the values of p and θ, given that $\cos(p+2)\theta + \cos p\theta = \cos\theta$ for $0^o \le \theta \le 360^o$.

49. Express $10\sin x\cos x + 12\cos 2x$ in the form $R\sin(2x+\alpha)$, hence or otherwise solve $10\sin x\cos x + 12\cos 2x + 7 = 0$ in the range $0^o \le x \le 360^o$.

50. Prove that: $\cos^2 2A + \cos^2 2B + \cos^2 2C - 1 = 2\cos 2A\cos 2B\cos 2C$.

GEOMETRY

51. The equation of a circle is given by $x^2 + y^2 + Ax + By + C = 0$, where A, B, C are constants, given that $8A = 6B,\ 6A = 4C$ and $C = 18$, find the coordinates of the centre and the radius.

52. Prove that the line $2x - 3y + 26 = 0$ is a tangent to the circle $x^2 + y^2 - 4x + 6y - 104 = 0$ and hence find the coordinates of the point of intersection.

53. Prove that the circles $x^2 + y^2 - 6x - 12y + 40 = 0$ and $x^2 + y^2 - 4y = 16$ are orthogonal.

54. A point P on the curve is given parametrically by $x = 3 - \cos\theta$ and $y = 2 + \sec\theta$.

Find the: (i) equation of the normal to the curve at the point $\theta = \dfrac{\pi}{3}$

(ii) Cartesian equation of the curve.

(b) Point $P(ap^2,\ 2ap)$ and $Q(aq^2,\ 2aq)$ lie on the parabola $y^2 = 4ax$. Find the locus of the midpoint of the chord PQ for which $pq = 2a$.

55. Find the equation of the normal to the curve $y^2 = 4bx$ at the point $P(bp^2,\ 2bp)$. Given that the normal meets the curve again at $Q(bq^2,\ 2bq)$, prove that $p^2 + pq + 2 = 0$.

56. Show that the curve $x = 5 - 6y + y^2$ represents a parabola and find the directrix and sketch.

57. The normal to the parabola $y^2 = 4ax$ at the point $P(at^2,\ 2at)$ meets the axis of the parabola at G and GP is produced, beyond P to Q so that $\overline{GP} = \overline{PQ}$. Show that the equation of locus of Q is $y^2 = 16a(x+2a)$.

58. The points $P\left(5p,\ \frac{5}{p}\right)$ and $Q\left(5q,\ \frac{5}{q}\right)$ lie on the rectangular hyperbola $xy = 25$.

i) Find the equation of the tangent at P and hence deduce the equation of the tangent at Q.

ii) The tangents at P and Q meet at point N, show that the coordinates of N are $\left(\frac{10pq}{p+q},\ \frac{10}{p+q}\right)$.

VECTORS

59i) Show that the point with position vector $\mathbf{i} - 9\mathbf{j} + \mathbf{k}$ lies on the line $\mathbf{r} = 3\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 6\mathbf{j} - \mathbf{k})$.

ii) Show that the line $\frac{x-2}{2} = \frac{2-y}{1} = \frac{3-z}{-3}$ is parallel to the plane $4x - y - 4z = 0$.

iii) Find the shortest distance from the point with position vector $4\mathbf{i} - 3\mathbf{j} + 10\mathbf{k}$ to the line $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.

60i) A line passes through the mid point of $A(4,\ 3,\ 2)$ and $B(6,\ 2,\ 1)$. This line is parallel to the plane $6x + 3y + 9z = 11$. Find the equation of the line.

ii) A line passes through the point $P(1,\ -3, -4)$ and is perpendicular to the plane $-4x + 3y + 6z = 10$. If this line meets another plane $6x + 8y + 4z = 11$, find the point of intersection.

61. Two lines have vector equations $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = (4-\beta)\mathbf{i} + (\beta+4)\mathbf{j} + (1+2\beta)\mathbf{k}$. Find the position vector of the point of intersection of the two lines and the Cartesian equation of the plane containing the two lines.

62. A plane, P_1 passing through the point $A(1,\ 1,\ -3)$ is perpendicular to the line $\frac{x}{2}=\frac{1-y}{3}=\frac{z-3}{2}$. Another plane, P_2 contains the points with position vectors $\boldsymbol{p}=\boldsymbol{i}-2\boldsymbol{j}+5\boldsymbol{k}$, $\boldsymbol{q}=-2\boldsymbol{i}-4\boldsymbol{j}+3\boldsymbol{k}$ and $\boldsymbol{r}=7\boldsymbol{i}+3\boldsymbol{j}-8\boldsymbol{k}$. Find

(i) The Cartesian equations of the two planes.
(ii) The angle between the two planes.
(iii) The line of intersection of the two planes.

63. The position vectors of points A and B are **a** and **b** respectively. The point C is such that $\boldsymbol{BC}=\boldsymbol{a}$. Another point D on BC produced is such that AC intersects **OD** at T and $\boldsymbol{OT}:\boldsymbol{OD}=2:3$ with respect to the origin O.

a) Find in terms of **a** and **b**, the position vectors of (i) **BT** (ii) **TD.**

b) If $\boldsymbol{a}=15\boldsymbol{i}+\mathbf{12}\boldsymbol{j}+18\boldsymbol{k}$ and $\boldsymbol{b}=-10\boldsymbol{i}+16\boldsymbol{j}+6\boldsymbol{k}$ find to the nearest degree, the angle between **BT** and **TD.** Hence, find the area of triangle BTD.

64a) Find the equation of the plane that is perpendicular to line AB, and contains the point P , such that $AP:PB=2:1$, given that $A(1,0,-5)$ and $B(7,6,7)$.

b) The L_1 passes through the point $P(2,0,-4)$ and is parallel to the line $r=\begin{pmatrix}4\\-3\\1\end{pmatrix}+\mu\begin{pmatrix}2\\1\\2\end{pmatrix}$, Determine

i) the position vector of F, the point of intersection of the plane in (a) above and the line L_1

ii) The shortest distance between $P(2,0,-4)$ and the plane in (a).

iii) $\tan\theta$ where θ is the angle between the line L_1 and the plane in (a).

65a) Find the equation of the plane containing the lines $\frac{x-2}{2}=\frac{y+3}{3}=\frac{z-1}{5}$ and $r=\begin{pmatrix}5\\1\\3\end{pmatrix}+\mu\begin{pmatrix}8\\12\\20\end{pmatrix}$.

b) Find the point of intersection of the line $\frac{x-2}{2}=\frac{y+3}{3}=\frac{z-1}{5}$ and the line given by the equation $\boldsymbol{r}=(4+2t)\boldsymbol{i}+(8-5t)\boldsymbol{j}+(7+4t)\boldsymbol{k}$.

ii) Determine the angle between the two lines in (b) above. **END**

OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL PURE MATHEMATICS SEMINAR QUESTIONS 2023

ALGEBRA

1. (a) Find the square root of $14 + 6\sqrt{5}$

(b) Given $y = x + \frac{1}{x}$, Solve for x in the equation $4x^4 + 17x^3 + 8x^2 + 17x + 4 = 0$

(c) Find the first three terms of the Binomial expansion of $\sqrt{(1+x)(1+x^2)}$

(d) Given that the equation $x^2 + 3x + 2 = 0$ has roots k and l, find the equation whose roots are $\frac{k}{l^2}$ and $\frac{l}{k^2}$

(OLAN)

2. (a) Prove by induction that for all positive integer$\sum_{r=1}^{n}(3r+1)(r+2) = n(n+2)(n+3)$

(b) Prove by induction that for all positive odd integers, n, $f(n) = 4^n + 5^n + 6^n$ is divisible by 15

(c) Use Demoivre's theorem to prove that $sin5\theta = 5sin\theta - 20sin^3\theta + 16sin^5\theta$

(d) Hence or otherwise, find the distinct roots of the equation $2 + 10x - 40x^2 + 32x^5 = 0$ giving your answer correct to 3 dps where appropriate. **(JINJA COLLEGE)**

3. (a) The sum of n terms of the sequence is $S_n = 2^{2n} - n$ where n a natural numbers is 61. Find the first three terms of the sequence.

(b)From a class of 14 boys and 10 girls, 10 students are to be selected for a competition in which 5 boys and 5girls or 2girls and 8boys are to go for it. In how many ways can they be selected?

(c) The roots p and q of a quadratic equation are such that $p^3 + q^3 = 4$ and $pq = \frac{1}{2}(p^3 + q^3) + 1$. Find a quadratic equation with integral coefficients whose roots are p^6 and q^6.

(d). Mary operates an account with a bank which offers a compound interest of 5% per annum. She opened the account at beginning of 2019 with shs 800,000 and continue to deposit the same amount at beginning of every year. How much will she receive at the end of 2022. If she made no withdrawal with in this period?

(ST CHARLES LWANGA BUKERERE)

4. (a) Solve the inequality $\frac{x+3}{x-2} \geq \frac{x+1}{x-2}$

(b) Given the curve $y = \frac{x^2+x-2}{x^3-7x^2+14x-8}$

(i) Give the coordinates of the hole

(ii) Find the equations of the asymptotes

(iii) Determine the turning points and their nature

(iv) Find the intercepts and sketch the curve.

(ST MICHEAL S.S SONDE)

TRIGONOMETRY

5. (a) Given the function, $f(x) = \frac{3}{13+6sinx-5cosx}$ use the substitution $t = tan\left(\frac{x}{2}\right)$, to show that $f(x)$ can be written in the form $\frac{3(1+t^2)}{2\,(3t+1)^2+6}$

(b) Given that $cot^2\theta + 3cosec^2\theta = 7$, show that $tan\theta = \pm 1$

(c) (i) Express the function $y = 3cosx - \sqrt{3}sinx$ in the form $Rcos(\theta + \alpha)$where R is a constant and $0 \le \alpha \le 2\pi$

(ii) Hence find the coordinates of the minimum point of y

(iii) State the values of x at which the curve cuts the $x - axis$

(CODE HIGH SCHOOL)

6. (a) Show that $cos3\theta = 4cos^3\theta - 3cos\theta$. Hence if $cos\theta = \frac{1}{2}(a + \frac{1}{a})$, prove that $cos3\theta = \left(a^3 + \frac{1}{a^3}\right)$

(b) Given that A, B, and C are angles of a triangle, prove that; $Sin^2 {}^A/_2 + Sin^2 {}^B/_2 + Sin^2 {}^C/_2 = 1 - 2Sin {}^A/_2\, Sin {}^B/_2\, Sin {}^C/_2$

(c) Given that $Sin(\theta + \alpha) = a$ and $Sin(\theta + \beta) = b$. Show that $Cos2(\alpha - \beta) - 4abCos(\alpha - \beta) = 1 - 2a^2 - 2b^2$.

(JINJA PROGRESSIVE)

7. (a) Given that $cos45^0 = \frac{1}{\sqrt{2}}$. Show without using a calculator or tables that $Sin\left(292\frac{1}{2}^0\right) = -\frac{1}{2}\sqrt{2+\sqrt{2}}$

(b) If $cos\alpha - cos\beta = \frac{2}{3}$ and $sin\alpha - sin\beta = \frac{5}{6}$, find the value of;

(i) $Sin\frac{1}{2}(\alpha + \beta)$ (ii) $Cos(\alpha + \beta)$

I(i) Given that A,B and C are angles of $\frac{a^2+b^2-c^2}{a^2-b^2+c^2} = tanBcotC$.

(ii) Find all the sides of a triangle ABC whose area is $1008cm^2$ and $a = 65cm,\ b + c = 97cm$.

(MENTHA HIGH SCHOOL)

8. (a) Given that X, Y, Z are angles of a triangle, Prove that $tan\left(\frac{X-Y}{2}\right) = \left(\frac{x-y}{x+y}\right) Cot\left(\frac{Z}{2}\right)$, hence solve the triangle if $x = 9cm, y = 5.7cm\ and\ Z = 57^0$.

(b) Prove that $\frac{Cos11^0+sin11^0}{Cos11^0-Sin11^0} = tan56^0$

(c) In triangle ABC, $AB = x - y, BC = x + y\ and\ CA = x$ Show that $CosA = \frac{x-4y}{2(x-y)}$

(d)(i) Show that $tan^{-1}\left(\frac{1}{2}\right) + tan^{-1}\left(\frac{1}{5}\right) = tan^{-1}\left(\frac{7}{9}\right)$.

(ii) Prove that $Sin(2sin^{-1}x + cos^{-1}x) = \sqrt{1-x^2}$

(GAYAZA ROAD TRIANGLE KIWENDA)

ANALYSIS

9. (a) Use the method of small changes to find the value of $\frac{1}{\sqrt{0.97}}$ correct to 3dps

(b) Evaluate $\int_0^1 \frac{8x-8}{(x+1)^2(x-3)^2}\,dx$

(c)(i) Given that $f(x) = \frac{x^4+x^3-6x^2-13x-6}{x^3-7x-6}$, Express $f(x)$ into partial fractions

(ii) Hence evaluate $\int_4^5 f(x)\,dx$

(d) Evaluate $\int_0^{\frac{\pi}{4}} \frac{4}{1+cos2x}\,dx$

(MUKONO KINGS)

10(a)(i). On the same axes, sketch the curve $y = x(x+2)$ and $y = x(4-x)$.

(ii). Find the area enclosed by the two curves in $a(i)$ above

(iii). Determine the volume generated when the area enclosed by the two curves in $a(i)$ above is rotated about the $x-axis$.

(b) Evaluate $\int_2^6 \frac{\sqrt{x-2}}{x}\,dx$

(c) A match box consists of an outer cover open at both both ends into which a rectangular box without a top. The length of the box is 1.5 times the width. The thickness of the material is negligible and the volume of the match box is $25cm^3$. If the width is xcm, find interms of x the area of the material used. Hence show that if the least area of the material is to be used to make the box then the length should be approximately $3.7cm$

(OLAM)

11.(a) In order to post a parcel, the sum of the circumference of a cylindrical parcel and its height should add up to $6cm$. Find the dimensions of the largest parcel that can be accepted.

(b) A sample of bacteria in a sealed container is being studied. The number of bacteria, P in thousands, is given by the differential equation $(1+t)\frac{dp}{dt} + p = (1+t)\sqrt{t}$ where t the time in hours after the start of the study is. Initially, there are exactly 5,000 bacteria in the container.

(i) Determine, according to the differential equation, the number of bacteria in the container $8hours$ after the start of the study.

(ii) Find, according to the differential equation, the rate of change of the number of bacteria in the container 4 $hours$ after the start of the study.

(NAMRUTH HIGH SCHOOL)

12. (a) Given that $y = \log_e \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$, prove that $\frac{dy}{dx} = -secx$

(b) Solve the differential equation $x\frac{dy}{dx} = 2x - y$

(c) In an agricultural plantation the proportion of the total area that has been destroyed by a bacterial disease is x. The rate of the destruction of the plantation is proportional to the product of the proportion already destroyed and that not yet. It was initially noticed that half of the plantation had been destroyed by the disease and that at this rate another quarter of the plantation would be destroyed in the next $6 hours$.

(i) Form a differential equation relating x and time t

(ii) Calculate the percentage of the population destroyed 12 $hours$ after the disease was noticed.

(ST JAMES BUDDO)

VECTORS

13. (a) Find the angle $\alpha = \langle BAC$ of the triangle ABC whose vertices are $A(1,\ 0,\ 1)$, $B(2, -1,\ 1)$ and $C(-2,\ 1,\ 0)$.

(b) The planes $P_1 and\ P_2$ are respectively given by the equations $r = 2i + 4j - k + \lambda(i + 2j - 3k) + \mu(-i + 2j + k)$ and $r.(2i - j + 3k) = 5$ where λ and μ are scalar parameters. Find;

(i) The Cartesian equation for plane, P_1 .

(i) To the nearest degree, the acute angle between P_1 and P_2

(iii) The coordinates of the point of intersection of the plane, P_1 and the line $\frac{x-1}{5} = \frac{y-3}{-3} = \frac{z+3}{4}$

(WAKISO HILLS)

14(a) Given the line $r = (3 + 2\mu)i + (1 - \mu)j + (-2 + 2\mu)k$. Find the;

(i) Value of d if the line is in the plane $r.(i - 2j - 2k) = d$.

(ii) Distance of the point (3,1,7) from the line.

(b) Given the points $A(2, -5, 3)\ and\ B(7, 0, -2)$, find the coordinates of point C which divides AB externally in the ratio 3: 8.

(c) Show that the points $P(1,2,3), R(3,8,1)\ and\ T(7,20,-3)$ are collinear.

(PRIDE COLLEGE)

15(a) Find the equation of the plane which contains a point $A(2,1,-2)$and is parallel to the plane $x - y - 4z = 3$

(b) If a line $\frac{x}{2} = \frac{y+3}{0} = \frac{z-1}{1}$ intersects with the plane in (a) above, find the;

(i) Point of intersection.

(ii) The angle between the line and the plane.

(WHITE ANGELS HIGH SCHOOL)

16 (a). Find the vector equation of the line of intersection between the planes $r.\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} = 6$ and $r.\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 4$.

(b) Using the dot product only, find the equation of the plane containing points $A(0,1,1), B(2,1,0)$ and $C(-2,0,3)$.

(c) A straight line joining the points $(2,1,4)$ and $(a-1,4,-1)$ is parallel to the line joining points $(0,2,b-1)$ and $(5,3,-2)$. Find the values of a and b.

(TALENTS COLLEGE)

COORDINATE GEOMETRY

17.(a) The points A, B, and C have coordinates $A(-3,2)$, $B(-1,-2)$ and $C(0,n)$, where n is a constant. Given that $\overline{BC} = \frac{1}{5}\overline{AC}$, Find possible values of n.

(b) A straight line L, passes through the point (-2, 1) and makes an angle of 45^0 with the horizontal.

(i) Find the equation of line L.

(ii) Given that the line L, intersects the $x-axis$ at A and the $y-axis$ at B. Find the distance AB.

(c) Find the coordinates of the circumcenter of the triangle ABC with vertices $A(3,2), B(1,4)$ and $C(5,4)$.

(d) Given that $A(0,-5), B(-7,2)$ and $C(2,11)$ are vertices of a parallelogram $ABCD$, find the coordinates of the point D.

(MAKERERE HIGH SCHOOL)

18.(a) Find the equation of the circle whose centre is at (5,4) and touches the line joining (0,5) and (4,1)

(b) A circle that passes through the points $A(3,4)$ and $B(6,1)$ and the equation of the tangent to this circle at A is the line $2y = x + 5$. Find;

(i) The coordinates of the centre of the circle.

(ii) The radius of the circle.

(iii) The equation of the circle.

(c) If $y = mx$ is a tangent to a circle $x^2 + y^2 + 2fy + c = 0$ prove that $c = \frac{f^2m^2}{1+m^2}$. Hence find the equation of the tangents from origin to the circle $x^2 + y^2 - 10y + 20 = 0$.

(d) Show that the circles whose equations are $x^2 + y^2 - 4y - 5 = 0$ and $x^2 + y^2 - 8x + 2y + 1 = 0$ cut orthogonally.

(QUEENS S.S)

19.(a) P and Q are two points whose coordinates are $(at^2, 2at)$, $(\frac{a}{t^2}, \frac{-2a}{t})$ respectively and S is a point$(a,0)$. Show that $\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}$

(b) Prove that $x = 3t^2 + 1$ and $2y = 3t + 1$ are parametric equations of a parabola. Find its Vertex, Focus and length of latus rectum.

(c) The point $P(at^2, 2at)$ is on the parabola $y^2 = 4ax$. The chord OQ passes through the origin$O$. The tangent at P is parallel to the chord OQ. The tangents to the parabola at P and Q meet at a point R. Determine the coordinates of points Q and R in terms of a and t.

(ST JOHNS NTEBETEBE)

20.(a) Show that $x^2 + 2y^2 + 6x - 8y = 7$ is an ellipse and hence determine its centre and eccentricity

(b) Points S and S' are the foci of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. Find the coordinates of S and S'

(c) The conic section below has eccentricity $e < 1$ and equation $\frac{x^2}{9} + y^2 = 1$. Find the value of e

(d).If the line $y = mx + c$ is a tangent to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $c^2 = b^2 + a^2m^2$.

Hence determine;

(i) Equations of the four common tangents to the ellipses $\frac{x^2}{23} + \frac{y^2}{3} = 1$ and $\frac{x^2}{14} + \frac{y^2}{4} = 1$

(ii) The equations of the tangents at the point $(-3,3)$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(PAUL MUKASA HIGH SCHOOL)

21(a). Find the equation of the tangent to the hyperbola $x^2 - 9y^2 = 1$ at $P(sec\beta, \frac{1}{3}tan\beta)$

(b) The tangent at any point $P\left(ct, \frac{c}{t}\right)$ on the hyperbola $xy = c^2$ meets x and y at A and B respectively. O is the origin.

(i) Prove that $AP = PB$

(ii) Prove that the area of triangle AOB is constant

(iii) If the hyperbola is rotated through an angle of -45^0 about O, find the new equation of the curve.

(GOOD CHOICE HIGH SCHOOL)

END

OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL PURE MATHEMATICS SEMINAR QUESTIONS 2022

ALGEBRA

1. (a) Without using mathematical tables or calculators, find the value of $\frac{(\sqrt{5}-2)^2-(\sqrt{5}+2)^2}{8\sqrt{5}}$.

(b) Given that $2x^2+7x-4$, x^2+3x-4, and $7x^2+ax-8$ have a common factor, find the;

(i) Factors of $2x^2+7x-4$ and x^2+3x-4.

(ii) Value of a in $7x^2+ax-8$.

(c) A man deposits shs 150,000 at the beginning of every year in a microfinance bank with the understanding that at the end of seven years, he is paid back his money with 5% per annum compound interest. How much does he receive?

(d) Given that the roots of the equation; $x^2-bx+c=0$ are $\sqrt{\propto}$ and $\sqrt{\beta}$. Show that;

(i) $\propto+\beta=b^2-2c$

(ii) $\propto^2+\beta^2=\left(b^2-2c-\sqrt{2}c\right)\left(b^2-2c+\sqrt{2}c\right)$

2. (a) Find the solution set for which $\log_2 x-\log_x 4\le 1$.

(b) Solve the simultaneous equations; $2a-3b+c=10$,
$a+4b+2c+3=0$,
$5a-2b-c=7$

(c) A geometric progression has the first term 10 and sum to infinity of 12.5. How many terms of the progression are needed to make a sum which exceeds 10?

(d) Given that the equations $y^3-2y+4=0$ and $y^2+y+c=0$ have a common root, show that $c^3+4c^2+14c+20=0$.

3. (a) Simplify $(2+5i)^2+5\left(\frac{7+2i}{3-4i}\right)-i(4-6i)$ expressing your answer in the form $a+bi$.

(b) The roots of the equation $3x^2+2x-5=0$ are $\propto$ and β. Find the value of $\propto^4+\beta^4$.

(c) Solve the equation $\sqrt{x+5}+\sqrt{x+21}=\sqrt{6x+40}$.

(d) Given that $\log_5 21=m$ and $\log_9 75=n$, show that $\log_5 7=\frac{1}{2n-1}(2nm-m-2)$

4. (a) Expand $(1-x)^{\frac{1}{3}}$ as far as the term in x^3. Use your expansion to deduce $\sqrt[3]{24}$ correct to three s.f.

(b) In the expansion of $(1+ax)^n$, the first three terms are $1-\frac{5}{2}x+\frac{75}{8}x^2$. Find n and a , state the range of values for which the expansion is valid.

(c) Determine the term independent of x in the binomial expansion of $\left(\frac{3}{x^2} - 2x\right)^6$.

(d) Find the coefficient of x^{17} in the expansion of $\left(x^3 + \frac{1}{x^4}\right)^{15}$

ANALYSIS

5. (a) If $y = tan^{-1}\left(\frac{ax-b}{bx+a}\right)$,show that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

(b) Differentiate $cos(x^2 e^x)$ with respect to x.

(c) Differentiate $cos^2 x$ with respect to x from first principles.

(d) Given that $x = \frac{3t-1}{t}$ and $y = \frac{t^2+4}{t}$,

(i) Show that $\frac{d^2 y}{dx^2} = 2t^3$

(ii) Determine the area of the largest rectangular piece of land that can be enclosed by $100m$ of fencing if part of an already existing wall is used.

6. (a) Solve the differential equation $\frac{dy}{dx} + \frac{2xy}{x^2+1} - x = 0$

(b) The gradient of a certain curve is given by kx. If the curve passes through the point (2,3) and the tangent at this point makes an angle of $tan^{-1}(6)$ with the positive direction of the x-axis, find the equation of the curve.

(c) A research to investigate the effect of a certain chemical on a virus infection, the research revealed that the rate at which the virus population is destroyed is directly proportional to the population at that time. Initially , the population was p_o. At $t\ months$ later, it was found to be p.

(i) Form a differential equation connecting p and t

(ii) Given that the virus population reduced to one third of the initial population in $4\ months$, solve the equation in c(i) above.

7. (a) Partialise fully, $f(x) = \frac{x^4+x^3-6x^2-13x-6}{x^3-7x-6}$. Hence $\int f(x)dx$ from 4 to 5.

(b)(i) On the same axes, sketch the curve $y = x(x+2)$ and $y = x(4-x)$

(ii) Find the area enclosed by the two curves in b(i) above

(iii) Determine the volume generated when the area enclosed by the two curves in b(ii) above is rotated about the x-axis.

8. (a) Find the integral $\int x cos^2 x\, dx$

(b)Find the area enclosed between the curve $y = x(x-1)(x-2)$ from $x = 0\ to\ x = 2$

(c) A conical vessel whose height is $10metres$ and radius of the base $5m$ is being filled with water at a uniform rate of $1.5m^3min^{-1}$. Find the rate at which the level of the water in the vessel is rising when the depth is $4m$

(d) Find the area enclosed by the curve $y = x - \frac{1}{x}$, the $x - axis$ and the line $x = 2$.

TRIGONOMETRY

9. (a) Solve the equation $3cos4\theta + 7cos2\theta = 0\ for\ 0^0 \leq \theta \leq 180^0$

(b) Express $10sinxcosx + 12\ cos2x$ in the form $Rsin(2x{+}\propto)$. Hence find the maximum value of $10sinxcosx + 12\ cos2x$.

(c) Prove that $\frac{cos11^0+sin11^0}{cos11^0-sin11^0} = tan56^0$

(d) In any triangle ABC, prove that $SinB + SinC - SinA = 4cos\, {}^A/_2\, Sin\, {}^B/_2\, Sin\, {}^C/_2$.

10. (a) Find the values of x lying between $-180^0\ and\ 180^0$ that satisfy the equation $10sin^2x + 10sinxcosx = cos^2x + 2$

(b) If $\frac{Sin16\theta cos2\theta - cos6\theta sin12\theta}{cos4\theta cos2\theta + sin6\theta sin8\theta} = tanm\theta$, where m is a constant, find the value of m.

(c) Find all the solutions to $2sin3\theta = 1$ for θ between $0^0\ and\ 360^0$. Hence find the solutions for $8x^3 - 6x + 1 = 0$

(d)Show that in any triangle ABC, $\frac{a^2-b^2}{c^2} = \frac{Sin(A-B)}{Sin(A+B)}$

11. (a) Using the t-formulae or otherwise prove that $1 + sec2\theta = tan2\theta cot\theta$

(b) Given that $y = \frac{sinx - 2sin2x + sin3x}{sinx + 2sin2x + sin3x}$;

(i) Prove that $y + tan^2\left(\frac{x}{2}\right) = 0$,

(ii) And hence express the exact value of $tan^2 15^0$ in the form $p + q\sqrt{r}$ where $p, q\ and\ r$ are integers.

(iii) Hence, find the value of x in the range $0^0 \leq x \leq 360^0$ for which $2y + sec^2\left(\frac{x}{2}\right) = 0$

12. (a)Given that $45^0 = \frac{1}{\sqrt{2}}$. Show without using a calculator or tables that $Sin\left(292\frac{1}{2}^0\right) = -\frac{1}{2}\sqrt{2 + \sqrt{2}}$

(b) Given that $P = 2cos2x + 3cos4x$ and $q = 2sin2x + 3sin4x$;

(i) Find the greatest and least value of $p^2 + q^2$.

(ii) Given that $p^2 + q^2 = 19$, find x for $0^0 \leq x \leq 90^0$

(iii) Using the result in (ii) above, and without using a calculator or Mathematical tables, show that $pq = \frac{-5\sqrt{3}}{4}$.

VECTORS

13. (a) Line A is intersection of two planes whose equations are $3x - y + z = 2$ *and* $x - 5y + 2z = 6$. Find the Cartesian equation of the line

(b) Given that line B is perpendicular to the plane $3x - y + z = 2$ and passes through the point C(1,1,0), find the;

(i) Cartesian equation of the line B

(ii) Angle between the line B and line A in (a) above.

(c) The point $A(2,-1,0)$, B(-2, 5, -4) and C are on a straight line such that $3\overrightarrow{AB} = 2\overrightarrow{AC}$. Find the coordinates of C.

14. (a)The points P, Q, R have position vectors $2\boldsymbol{a} - 5\boldsymbol{b}, 5\boldsymbol{a} - \boldsymbol{b}, and\ 11\boldsymbol{a} + 7\boldsymbol{b}$ respectively. Show that P, Q and R are collinear and state the ratio $PQ: QR$.

(b) Calculate the perpendicular distance from the point $(1, -2, 3)$ from the line with equation $\boldsymbol{r} = 2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k} + t(2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k})$.

(c) Determine the equation of a plane through the point $(1, -3, 2)$ and contains the vectors $\boldsymbol{i} - 3\boldsymbol{j} + 3\boldsymbol{k}$ and $-\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$.

15. (a) The point $C(a, 4, 5)$ divides the line joining $A(1,2,3)$ and B (6,7,8) in the ratio $\lambda: 3$. Find a and λ

(b) Find the equation of a plane which contains the line $\frac{x-1}{2} = \frac{y+4}{-3} = \frac{z+1}{-1}$ and passes through the point $(2, 3, -1)$

(c) Show that the lines L_1, with vector equation $\boldsymbol{r} = \binom{2}{5} + \lambda\binom{2}{-3}$ and L_2, vector equation $r = \binom{3}{-3} + \mu\binom{3}{2}$ are perpendicular and find the position vector of their point of intersection.

16. (a) Find the length of the perpendicular distance from $A(4,3,5)$ to the plane $6x - y + 2z = 14$

(b) Find the foot of the perpendicular drawn from the point $(2, -1, 5)$ to the line $\frac{x-11}{10} = \frac{y+2}{-2} = \frac{z+5}{-11}$.

(c) Find the angle between the plane $x - 2y + z = 20$ and the line $\frac{2-x}{-3} = \frac{y+1}{4} = \frac{2-z}{-12}$

(d) $PQRS$ is a quadrilateral $P(1,-2), Q(4,-1), R(5,2)$ and $S(2,1)$. Show that the quadrilateral is a rhombus.

COORDINATE GEOMETRY

17. (a) Find the locus of a point which moves such that the ratio of its distance from the point $A(2,4)$ to its distance from the point $B(-5,3)$ is $2:3$

(b) A point P moves such that its distance from two points $A(-2,0)$ and $B(8,6)$ is in the ratio $AP:PB = 3:2$. Show that the locus of P is a circle.

(c) Find the locus of the point $P(x,y)$ which moves such that its distance from the point $A(5,3)$ is twice its distance from $x = 2$

(d) Find the centroid of the triangle whose sides are given by the equations $x + y = 11$, $y = x - 1$ and $3y = x - 3$.

18. (a) find the equation of the circle whose end diameter is the line joining the points $A(1,3)$ and $B(-2,5)$.

(b) A circle whose centre is in the first quadrant touches the x- and y-axes and the line $8x - 15y = 20$.

Find the;

(i) Equation of the circle

(ii) Point at which the circle touches the y-axis

(c) If the x-axis and y-axis are tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, Show that $c = g^2 = f^2$

19. (a) ABCD is a square inscribed in a circle $x^2 + y^2 - 4x - 3y = 36$, Find the length of diagonals and the area of the square

(b) Find the coordinates of the foot of the perpendicular from the point (2,-6) to the line $3y - x + 2 = 0$.

(c) A circle touches both x-axis and the line $4x - 3y + 4 = 0$. Its centre is in the first quadrant and lies on the line $x - y - 1 = 0$. Prove that its equation is $x^2 + y^2 - 6x - 4y + 9 = 0$

(d) ABCD is a rhombus such that the coordinates $A(-3,-4)$ and $C(5,4)$. Find the equation of the diagonal BD of the rhombus. If the gradient of side BC is 2 obtain the coordinates of B and D, Prove that the area of the rhombus is $21\frac{1}{3} sq.Units.$

20. (a) Show that the equation $y^2 - 4y = 4x$ represents a parabola; hence determine the focus, vertex and directrix.

(b) A tangent from the point $T(t^2, 2t)$ touches the curve $y^2 = 4x$. Find;

(i) The equation of the tangent

(ii) The equation of the line L parallel to the normal at $T(t^2, 2t)$ and passes through (1,0)

(iii) The point of intersection, X, of the line L and the tangent

(c) A point $P(x,y)$ is equidistant from X and T in (b) above, Show that the locus of P is $t^3 + 3t - 2(xt + y) = 0$

(d) Find the equations of the tangents to the parabola $y^2 = 6x$ which pass through the point (10,-8)

END